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Multistability in anisotropic textured shells

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1. Introduction

Structures with many stable states can provide a key underlying technology for reconfigurable smart systems. By introducing local indentation in a shell, and combining this texture with prestress, a range of interesting multistable properties can be achieved. Textured shells have small surface features at a scale intermediate between the overall structural scale, and the material thickness. This study will focus on a particular class of novel textured shell surfaces and investigate the effective anisotropy for overall bending behaviour of the shell structure.

Previously, it has been shown that cylindrically curved shells can show interesting multistable behaviour. These shell structures can be given a second cylindrical stable configuration either by using a non-isotropic material (Guest & Pellegrino [3]) or by imposing prestress on an isotropic shell (Kebadze et al. [4]).

The idea of forming surface features in the shell and combining this effective anisotropy with prestressing has previously been introduced by Golabchi & Guest [2]. These novel prestressed textured shells provide a great variety of possibilities for the design of multistable structures, where the structure can be repeatably transferred between stable configurations. They can be used as components of a morphing multistable structure: for instance they might be used in deployable structures, or in reconfigurable aircraft.

2. Bistable prestressed textured shells

Small surface features, or *texture*, have been used to replace the isotropic behaviour of the shell structure. Taking advantage of the anisotropic properties of the textured shell and further prestressing the structure, can lead to a range of distinctive multistable properties. For instance, different interesting bistable behaviour can be engendered by rolling a flat textured shell consecutively in orthogonal directions. In this plastic forming process, if the second rolling is in the same sense as the first, the shell shows *synclastic* bistable behaviour, however, the second rolling can be with opposite-sense bending, where the shell shows *anticlastic* bistable behaviour, see Figure 1.

Synclastic bistable behaviour   *Anticlastic* bistable behaviour

Figure 1: Textured shell; showing different bistable behaviour

In these textured surfaces, the residual stresses left by plastic rolling processes were essential to the bistability. Furthermore, the texture ensures that the global properties of the shell are no longer isotropic, which in particular would prevent the synclastic bistable case.

The textured surface has a significant effect on the overall behaviour of the shell structure, with the dimples leading to a non-isotropic global behaviour. This paper will explore the non-isotropic bending behaviour of the textured shell structure both computationally and experimentally.

3. Computational results

The bending properties of the textured shell have been studied using finite element simulations. The numerical studies were conducted on a shell where dimples are placed in the surface at regular intervals and the shape of the shell is approximated by a doubly-sinusoidal surface described as:

$$z = \frac{d}{2} . sin\left(\frac{2\pi}{\lambda} x\right) . sin\left(\frac{2\pi}{\lambda} y\right), \qquad (1)$$

where d is the depth, and λ is the wavelength of the corrugations. Based on the computational results, a simple analytical calculation is carried out to determine the relationships between the components of the bending stiffness matrix. This is important in considering bistability, as Guest and Pellegrino [3] have previously argued that bistability in cylindrical curved shells is dependent on the relative magnitudes of the component of the bending stiffness matrix; increasing the coupling between bending in two directions will tend to create bistability in the structure.

For an element of a shell a constitutive bending relationship is given by (see e.g. Calladine [1]):

$$\mathbf{M} = \mathbf{D} . \boldsymbol{\kappa} , \qquad (2)$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{21} & D_{22} & D_{26} \\ D_{61} & D_{62} & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} . \qquad (3)$$

In this particular example, the textured shell is orthotropic, and hence there is no coupling between bending and twisting ($D_{16} = D_{26} = 0$). By assuming that there is no moment along the transverse direction of the structure for a very narrow strip of the textured shell ($M_y = 0$), the bending stiffness of the corrugated shell in the x direction can be written as:

$$\frac{M_x}{\kappa_x} = D_{11} - \frac{{D_{12}}^2}{D_{11}} = \alpha \; . \tag{4}$$

Alternatively, for a wide strip of the textured shell, we can assume there is no curvature along the transverse direction of the shell ($\kappa_y = 0$); the bending stiffness is given by:

$$\frac{M_x}{\kappa_x} = D_{11} = \beta \; . \tag{5}$$

Therefore, the relative coupling between bending in two orthogonal directions can be expressed as:

$$\nu = -\frac{D_{12}}{D_{11}} = \pm \sqrt{1 - \frac{\alpha}{\beta}} \; . \tag{6}$$

Comparing the relative values of the bending stiffness derived from the FE simulation for a very narrow, α, and very wide, β, strips of the textured shell, enable us to illustrate the variation of the bending coupling term with corrugation angle, see Figure 2.
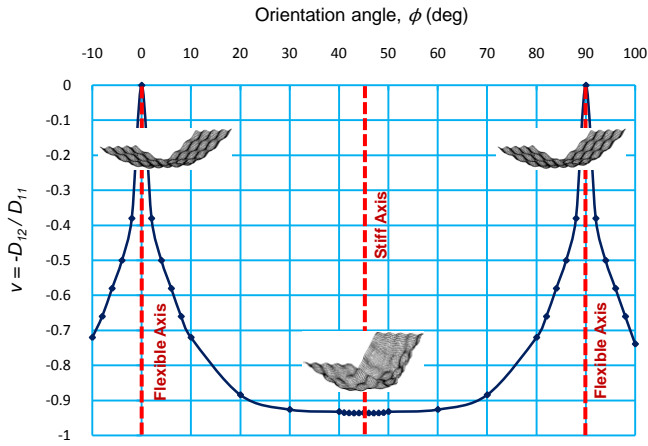

Figure 2: Variation of effective Poisson's ratio $\nu = -D_{12}/D_{11}$ with orientation of corrugation.

It can be seen that for bending about the flexible axis, $\phi = 0°$, the coupling between bending in two orthogonal directions is negligible; in this case the shell responds in *cylindrical* bending. However, as the corrugation angle increases, the coupling between bending in two directions increases rapidly and the shell tends to have a bowl shape deformation; Figure 3(b) depicts this *spatial* bending.
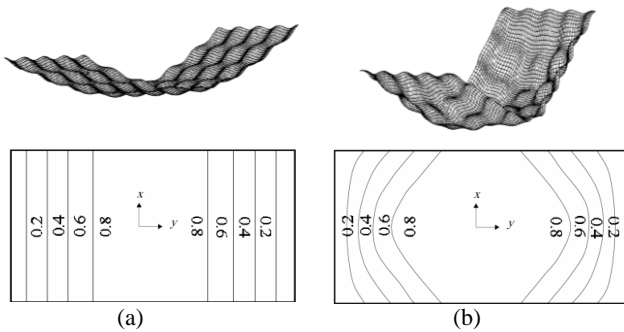

Figure 3: Bending behaviour of the textured shell (a) $\phi = 0°$, and (b) $\phi = 45°$; the deformed shape of the structure and a contour plot of the relative magnitude of the vertical deformation is shown.

Remarkably, the computational results in this study have shown that dimpled textured shells can achieve an effective Poisson's ratio in bending of close to -1. This significant Poisson's ratio shows a strong coupling between bending in two orthogonal directions and reinforces the potential of using these textured shells as multistable structures.

4. Experimental setup

A novel experiment has been set up to measure the bending stiffness of thin shells, to verify the computational results, and further explore how changes to the texture pattern can affect the bending behaviour of the shell surface. The particular test specimen for the current work is made of hardened Copper Beryllium formed by a punching process with a doubly corrugated surface, and is capable of a large change in shape without having plastic deformation; this would provide the closest correlation between experimental and computational results.

The bending test required the use of a special test arrangement, as a standard four point bending test is not suitable to examine large shape changing components. Therefore, a new bending test rig needed to be design which was capable of testing shells that undergo large change of curvature and could induce a uniform curvature along the shells.

The test is carried out using an Instron material testing machine with a 100 N load cell, which is connected to two rigid cylinders through several cables. The model is free along the long edges, fixed on the rigid cylinders along the short edges; as shown in Figure 4. Due to the symmetry in geometry and loading, equal rotation will be imposed to the two end of the specimen by the Instron, while measuring the required end moments. Therefore, the model will have a constant moment imposed along its length.

This experiment is designed in a way that there are no additional constraints in the test in order to prevent any unexpected load. The initial results have shown good correlation with the computational results.
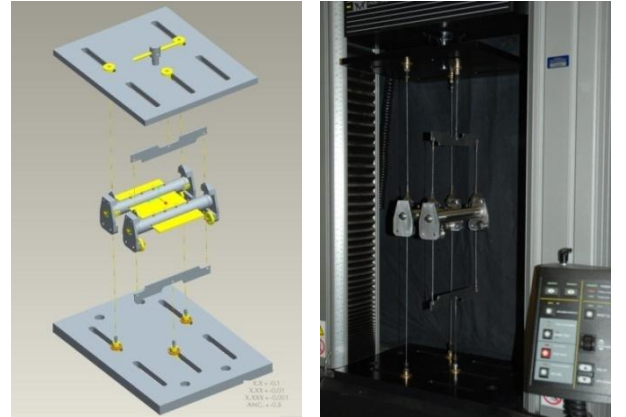

Figure 4: Experimental apparatus for the bending test

References

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