Engineering Tripos Part IB SECOND YEAR

**Part IB Paper 1: Mechanics**

**Examples Paper 1/0**

**Revision**



*The Part IB Mechanics Course introduces techniques for calculating accelerations and forces in machinery, which are further developments of techniques covered in Part IA. It is therefore important to be fully familiar with some of the Part IA material, in particular Velocity Diagrams and the application of Virtual Work in mechanisms.*

*More challenging questions (which you may want to discuss with your Supervisor) are marked* *.

## Velocity Diagrams

1. Figure 1 shows a mechanism in which crank OB of length 100 mm rotates about O at a constant angular velocity of 8 rad $s^{-1}$. The straight continuous link ABC is of length 350 mm and its ends are guided such that A moves horizontally and C moves vertically.

   (a) At the instant shown, construct the velocity diagram for the mechanism and hence find the velocities of A and C, and the sliding velocity at B. A suitable scale for the velocity diagram is to let 1 mm represent 10 mm $s^{-1}$.

   (b) If loads of 100N act at A and C in the directions shown, find the torque which must be applied to crank OB to maintain the motion. Neglect friction, and the masses of the moving parts.

   Hint: This problem is similar to the two more difficult quick-return mechanisms which you analysed in the Part IA laboratory on Velocities in Mechanisms – and cannot be solved by the straightforward procedure that you used for the easier one. A good approach here is to locate the Instantaneous Centre of the link AC, and use it to determine the *direction* of movement of the point on the link which is inside the slider at B. If you draw this line on your velocity diagram, you will find you have enough information to solve the problem.

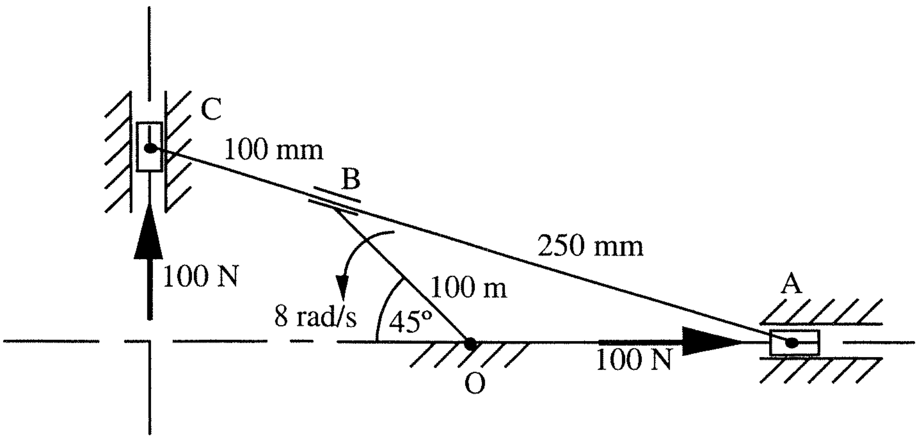


Fig. 1

2. (a) In the mechanism shown in Fig. 2, AB is fixed and AC rotates anti-clockwise with a uniform angular speed $\omega_{AC}$.

Sketch the velocity diagram for the mechanism in the position shown, and identify on it the angle which corresponds to $\phi$ on the space diagram. Hence, write down an expression for $\omega_{BD}$ in terms of $\omega_{AC}$, $\phi$, and any of the lengths AB, AC and BC. What is the angular velocity of the slider block C?

(b) The instantaneous centre of a moving body can be determined if either:

(i) the linear velocity of one point and the angular velocity are known, or

(ii) the directions of the velocities of two separate points on the body (extended, if necessary) are known.

Use one or other of these facts to locate the instantaneous centre of the slider block C.

* (c) Use your expression for $\omega_{BD}$ and the given dimensions of AB and AC to find the values of $\omega_{BD}$ when $\theta = 0°$ and $\theta = 180°$. Show that these are respectively the minimum and maximum values of $\omega_{BD}$ (it is much easier to demonstrate this geometrically than by algebra).

At what points in the mechanism's cycle does $\omega_{BD} = \omega_{AC}$ ?

* (d) Write down an expression for the velocity of sliding of the block C relative to the link BD in terms of $\phi$, and hence find the maximum sliding velocity and the values of $\theta$ when it occurs.

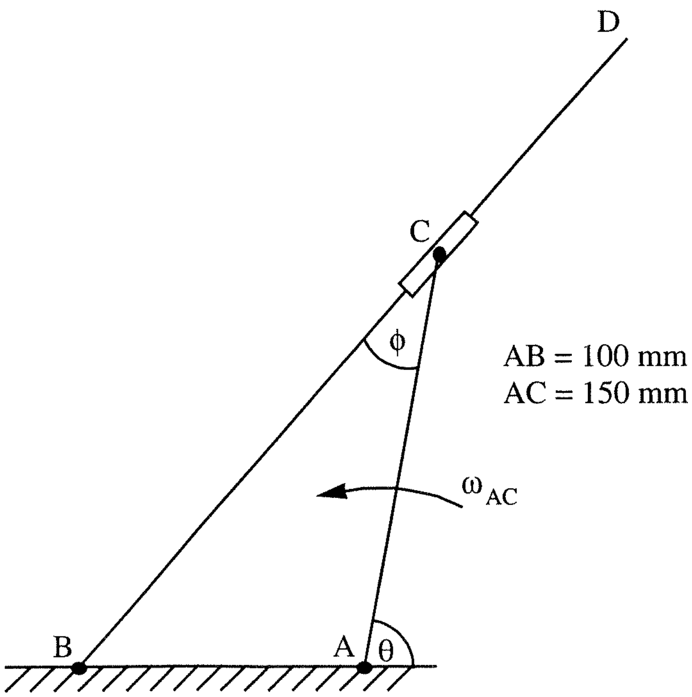


Fig. 2

**Vectors, Forces and Virtual Work**

3. A rigid body rotates with angular velocity $\omega \underline{e}$ about a fixed axis passing through the origin O of a Cartesian system of co-ordinates whose units are in metres, as shown in Fig. 3. The unit vector $\underline{e}$ has components $(1,1,1)/\sqrt{3}$, and the z-axis is vertical. A rigid link AB of length $\sqrt{30}$ is connected through a ball joint to the body at A. The other end of the link at B is connected through a second ball joint to a slider constrained to move along the z-axis. At a given instant, the joint A is located at (1,2,4).

   (a) Find the two possible locations of B.

   (b) Find the Cartesian components of the velocity of A, in terms of $\omega$.

   (c) Find the corresponding velocity of B, if it is in the higher of its two possible locations.

   (d) If the slider at B weighs 20 N and the rest of the mechanism is light and frictionless, find the torque required to rotate the rigid body when $\omega$ is small and positive.

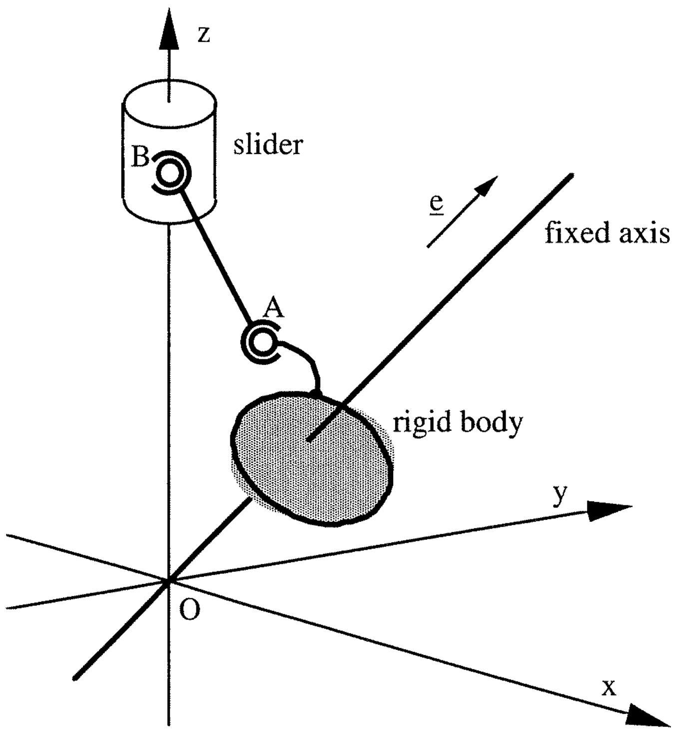


Fig. 3

4. (a) The line of action of a force ***P*** lies in the Oyz plane of a set of right-handed Cartesian axes, in which Oz is vertically up. ***P*** intersects the axis Oy at a distance $a$ from the origin and makes an angle of 30° with the Oz axis, as shown in Fig. 4. Determine:

(i) the component of ***P*** along the line x = y = z,

(ii) the moment of ***P*** about x = y = z.

* (b) A rotor of mass $M$ with axial symmetry has a tapped hole which mates with a fixed right-handed screw of pitch $\lambda$ whose axis coincides with the direction x = y = z. The rotor is maintained in equilibrium by the force ***P*** defined above.

Use the results from (i) and (ii) and the principle of virtual work to determine the magnitude of ***P***, assuming that there is no friction.

Hint: The *pitch* of a screw is the axial distance between adjacent threads. It is thus the distance the rotor would travel along the screw, if it was rotated by one complete revolution about the screw's axis.

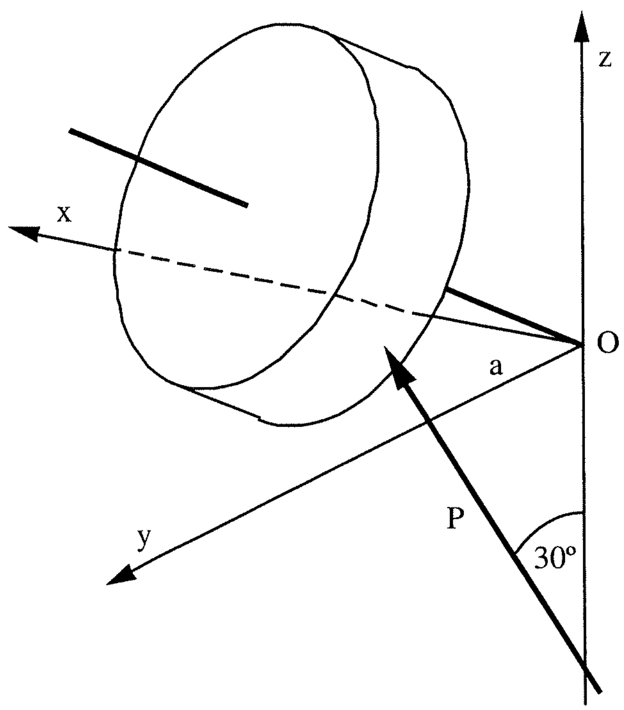


Fig. 4

## Accelerations in Polar Co-ordinates

5. An aircraft is being tracked by two radar stations A and B. Station B is 30 km to the East of A, and 20 km to the North of A. At a given instant, the aircraft is located due East of A, and the bearing from A to the aircraft is found to be altering at a constant rate of 0.01 rad $s^{-1}$ in an anticlockwise direction. At the same instant, the aircraft is detected to be due South of B, and the bearing from B to the aircraft is altering at a rate of 0.02 rad $s^{-1}$ in an anticlockwise direction; but this rate is increasing at a rate of $3 \times 10^{-4}$ rad $s^{-2}$. Unfortunately, atmospheric conditions prevent accurate measurement of the ranges of the aircraft from the two radar stations, or their derivatives.

   (a) What is the velocity (speed and direction) of the aircraft at this instant?

   (b) What are the Northerly and Easterly components of the aircraft's acceleration?

   (c) What is the instantaneous radius of curvature of the aircraft's path?

   Hint: you will need to use the information in Section 1.1 of the Mechanics Data Book, and calculate $\dot{r}$ for each radar station.

ANSWERS

1. (a) 330 mm $s^{-1} \rightarrow$ ; 1120 mm $s^{-1} \downarrow$ ; 700 mm $s^{-1}$ (b) 9.88 Nm anticlockwise

2. (a) $\dfrac{AC\,\omega_{AC}\cos\phi}{BC}$; the same as $\omega_{BD}$.

   (b) Instantaneous centre I is on AC (extended) such that angle CBI = $90^o$.

   (c) 0.6 $\omega_{AC}$ when $\theta = 0°$ and 3 $\omega_{AC}$ when $\theta = 180°$. When ABC = $90^o$.

   (d) 100 $\omega_{AC}$ mm $s^{-1}$, when $\theta = 131.8°$ or $228.2°$ (i.e. ABC = $90^o$, as above).

3. (a) (0, 0, 9) and (0, 0, -1) (b) $\dfrac{\omega}{\sqrt{3}}(2, -3, 1)$ (c) $\left(0,0,\dfrac{9\omega}{5\sqrt{3}}\right)$

   (d) 20.8 Nm

4. (a) (i) $\dfrac{P}{\sqrt{3}}\left(\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}\right)$ (ii) $\dfrac{aP}{2}$ (b) $P = \dfrac{Mg}{\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}+\dfrac{\sqrt{3}\pi a}{\lambda}}$

5. (a) 500 m $s^{-1}$ on a track of 053° (b) 8 m $s^{-2}$ and -6 m $s^{-2}$ (c) 25 km

AL Johnson
KM Wallace
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