PAPER 2: STRUCTURES REVISION PAPER

Q1

(a) $$\sigma_{max} = \frac{M}{Z_e}$$

$M = 300 \times 10^6$ Nmm

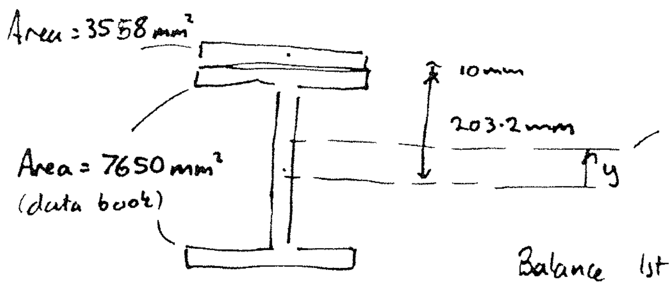
$Z_e = 1063$ cm^3 = 1063×10^3 mm^3

↑ from Data Book

$$\therefore \sigma_{max} = 282 \text{ N/mm}^2 \quad \text{(tensile)}$$

Similarly, $\sigma_{min} = -282$ N/mm^2 (compressive)

(b) Initially find new centroid



Balance 1st moment of areas

$$7650 \times y = 3558 \times (213.2 - y)$$

$$11208\, y = 758557$$

$$y = 67.7 \text{ mm}$$

Find I of total section

$$I_{total} = I_I + A_I \cdot y^2 + I_p + A_p \times (213.2 - y)^2$$

I_I (2nd moment of area of I-beam) = 21600 cm^4 (from data book)
= 216×10^6 mm^4

A_I (area of I beam) = 7650 mm^2

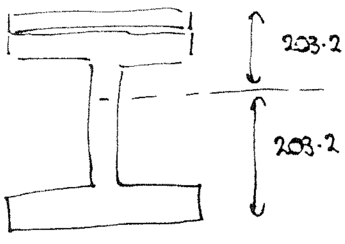
I_p (2nd moment of area of plate) = $\frac{bt^3}{12} = \frac{177.9 \times 20^3}{12}$
= 0.12×10^6 mm^4

A_p (area of plate) = 3558 mm^2

Q1

(b) (cont)

$$\therefore I_{total} = 216 \times 10^6 + 35 \times 10^6 + 0.12 \times 10^6 + 75 \times 10^6$$
$$= 326 \times 10^6 \text{ mm}^4$$

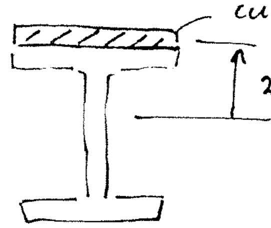


$$\sigma = \frac{My}{I}$$

$$\sigma_{bottom} = \frac{300 \times 10^6 \times 270.9}{326 \times 10^6} = 249 \text{ N/mm}^2$$
(tensile)

$$\sigma_{top} = \frac{300 \times 10^6 \times -155.5}{326 \times 10^6} = -143 \text{ N/mm}^2$$
(compressive)

(c) force / unit $= \frac{SA\bar{y}}{I}$



$$A\bar{y} = 3558 \times 145.5 = 517.7 \times 10^3 \text{ mm}^3$$

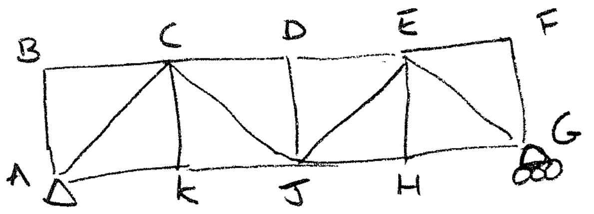
$$\therefore \text{force/unit length} = \frac{150 \times 10^3 \times 517.7 \times 10^3}{326 \times 10^6} = 2382 \text{ N/mm}$$

$$\text{Shear force / bolt} = 238.2 \text{ N/mm} \times \frac{200 \text{ mm}}{2} = \underline{\underline{23820 \text{ N}}}$$

2 bolts / 200 mm

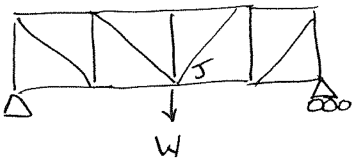
Q2

BAR	LOAD W AT J: Tension T × W	LOAD W AT J: Extension e × WL/EA	LOAD W* = 1 AT J: Tension T*	LOAD P* = 1 AT C: Tension T*
DJ	0	0	0	0
AK	0	0	0	0
AB	$-1/2$	$-1/2$	$-1/2$	$-3/4$
BK	$1/\sqrt{2}$	1	$1/\sqrt{2}$	$3/2\sqrt{2}$
BC	$-1/2$	$-1/2$	$-1/2$	$-3/4$
CK	$-1/2$	$-1/2$	$-1/2$	$-3/4$
KJ	$1/2$	$1/2$	$1/2$	$3/4$
CJ	$1/\sqrt{2}$	1	$1/\sqrt{2}$	$-1/2\sqrt{2}$
CD	-1	-1	-1	$-1/2$
HG	0	0	0	0
FG	$-1/2$	$-1/2$	$-1/2$	$-1/4$
FH	$1/\sqrt{2}$	1	$1/\sqrt{2}$	$1/2\sqrt{2}$
EF	$-1/2$	$-1/2$	$-1/2$	$-1/4$
EH	$-1/2$	$-1/2$	$-1/2$	$-1/4$
HJ	$1/2$	$1/2$	$1/2$	$1/4$
EJ	$1/\sqrt{2}$	1	$1/\sqrt{2}$	$1/2\sqrt{2}$
DE	-1	-1	-1	$-1/2$

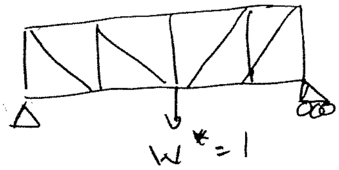


Q2 (continued)

b(i)



real compatible set



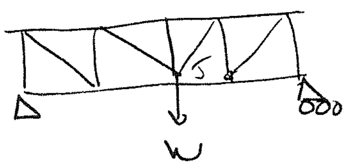
virtual equilibrium set

$$\Sigma T^* e = \left(\tfrac{1}{4} + \tfrac{1}{\sqrt{2}} + \tfrac{1}{4} + \tfrac{1}{4} + \tfrac{1}{4} + \tfrac{1}{\sqrt{2}} + 1\right) \times 2\, WL/EA$$
$$= \left(4 + 4/\sqrt{2}\right) WL/EA$$

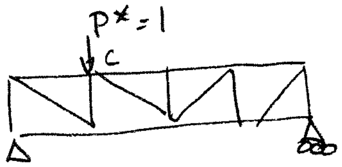
virtual work: $1 . \delta_{JV} = \Sigma T^* e$

$$\delta_{JV} = \left(4 + 4/\sqrt{2}\right) WL/EA \downarrow$$

(ii)



real compatible set



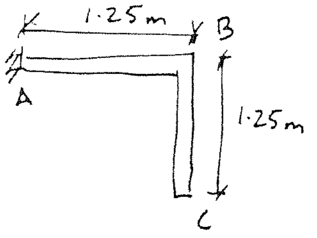
virtual equilibrium set

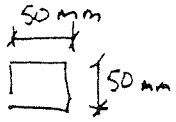
$$\Sigma T^* e = \left(\tfrac{3}{8} + \tfrac{3}{2\sqrt{2}} + \tfrac{3}{8} + \tfrac{3}{8} + \tfrac{3}{8} - \tfrac{1}{2\sqrt{2}} + \tfrac{1}{2} + \tfrac{1}{8} + \tfrac{1}{2\sqrt{2}} + \tfrac{1}{8} + \tfrac{1}{8} + \tfrac{1}{8} + \tfrac{1}{2\sqrt{2}} + \tfrac{1}{2}\right) \frac{WL}{EA}$$
$$= \left(3 + \sqrt{2}\right) WL/EA$$

virtual work: $1 . \delta_{CV} = \Sigma T^* e$

$$\delta_{CV} = \left(3 + \sqrt{2}\right) \frac{WL}{EA} \downarrow$$

Q3





$I = \frac{bd^3}{12} = 520833 mm^4$

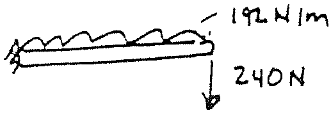
$E_{steel} = 210000 \ N/mm^2$

$\rho_{steel} = 7840 \ kg/m^3$

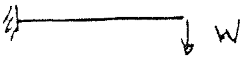
a) (i) vertical reaction at A

$V_A = \underset{m}{1.25} \times 2 \times \underset{m^2}{0.05^2} \times \underset{kg/m^3}{7840} \times \underset{m/s^2}{9.8} = 480.2 \ N$

(ii) self weight loads on span AB

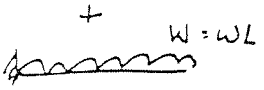


from Structures Data Book



$\delta_1 = \frac{WL^3}{3EI} = \frac{240 \times (1250)^3}{3 \times 210000 \times 520833} = 1.43 mm \downarrow$

+



$\delta_2 = \frac{WL^3}{8EI} = 0.54 mm \downarrow$

$\delta_{VB} = \delta_1 + \delta_2 = 1.43 + 0.54 = 1.97 mm \downarrow$

$\delta_{HB} = 0$

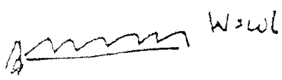
} Deflections of B

(iii)

end rotations due to loading

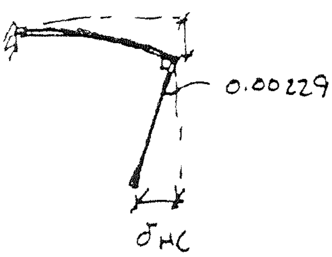
$\theta_1 = \frac{WL^2}{2EI} = \frac{240 \times (1250)^2}{2 \times 210000 \times 520833} = 0.00171 \ rad$

+



$\theta_2 = \frac{WL^2}{6EI} = 0.00057 \ rad$

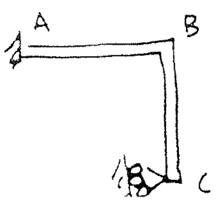
$\theta_B = \theta_1 + \theta_2 = 0.00229 \ rad$



$\delta_{HC} = \theta_B \cdot L = 0.00229 \times 1250 = 2.86 mm \leftarrow$

$\delta_{VC} = 1.97 mm \downarrow$

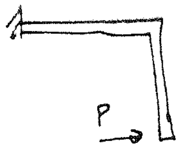
b)

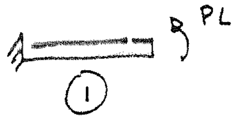


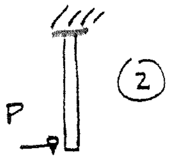
The structure is now statically indeterminate. Must consider compatibility.

Q3//

A horizontal force at C would result in the following displacements:





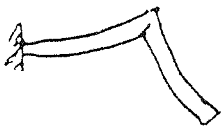


end rotation

$$\theta_B = \frac{ML}{EI} = \frac{PL^2}{EI}$$

end deflection

$$\delta_c = \frac{WL^3}{3EI} = \frac{PL^3}{3EI} \rightarrow$$



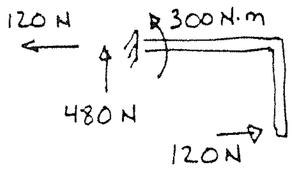
$$\delta_{HC} = \theta_B \cdot L + \delta_c = \frac{PL^3}{EI} + \frac{PL^3}{3EI} = \frac{4PL^3}{3EI}$$

For compatibility $\delta_{HC} = 0$

from before (part a iii) $\delta_{HC} = \left(\frac{WL^2}{2EI} + \frac{WL^2}{6EI}\right)L = \frac{2WL^3}{3EI}$

$$\therefore \frac{4PL^3}{3EI} = \frac{2WL^3}{3EI} \rightarrow P = \frac{W}{2} = 120\ N$$

Reactions at A + C



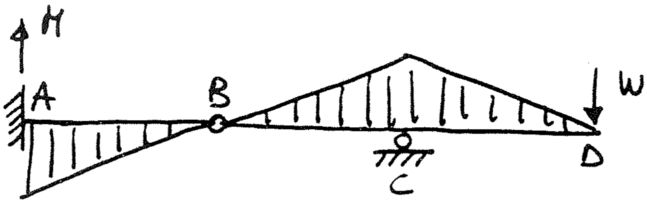
M_A

$$M_A + \frac{WL^2}{2} + WL - PL = 0$$

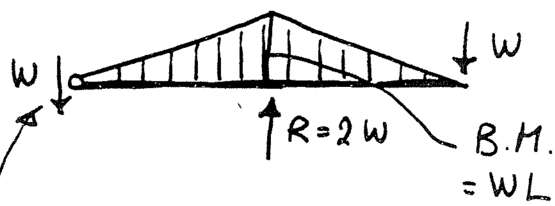
($\frac{WL^2}{2}$ = WL, P = W/2)

$$M_A = -WL = -240 \times 1.25 = -300\ N.m$$

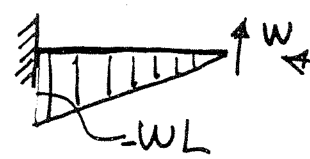
Q4//



Free body BCD:



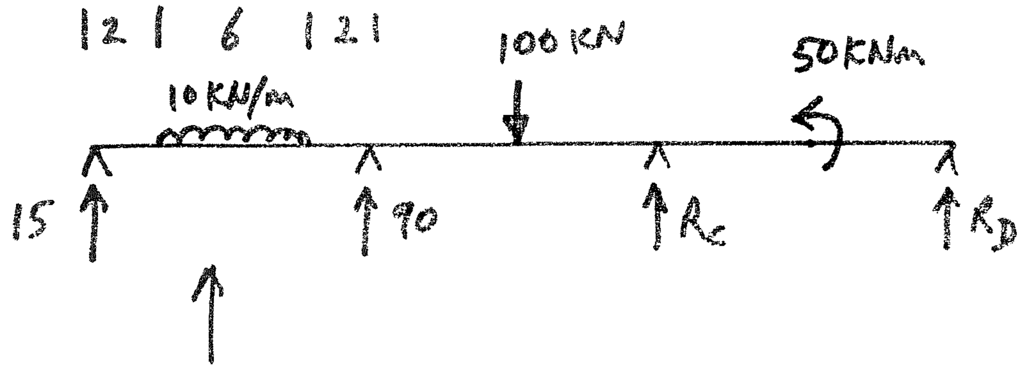
Free body AB:



shear force through pin.

Q5

(a)



Total load = 10 KN/m × 6 m = 60 KN

More than half of the candidates made this 80 KN!!

A beam on 4 supports would normally be statically indeterminate, but two of the support reactions have been given.

∴ R_C + R_D can be found by simple equilibrium

Resolve vertically

$$15 + 90 + R_C + R_D = 60 + 100$$

$$\Rightarrow R_C + R_D = 55 \text{ KN}$$

Take moments about D (to eliminate one of the unknowns)

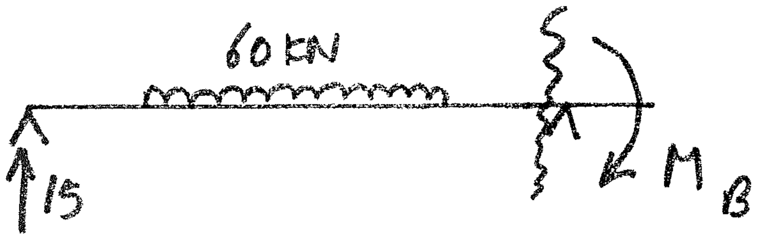
$$15.30 + 90.20 + R_C.10 = 60.25 + 100.15 + 50$$

$\Rightarrow$ **$R_C = 80$ KN ↑** **$R_D = -25$ KN** (downwards)

[5 marks]

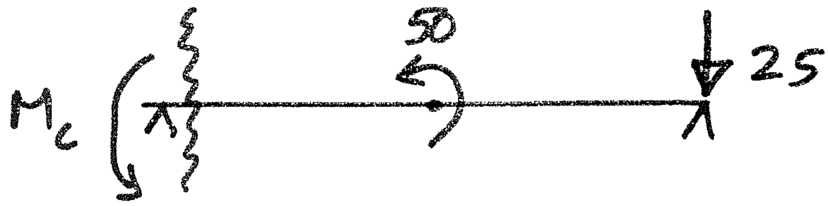
Q5

(b) Take a free body cut at B



$$M_B = 60 \times 5 - 15 \cdot 10 = \underline{\underline{150 \text{ KN m}}}$$

Take a free body cut at C



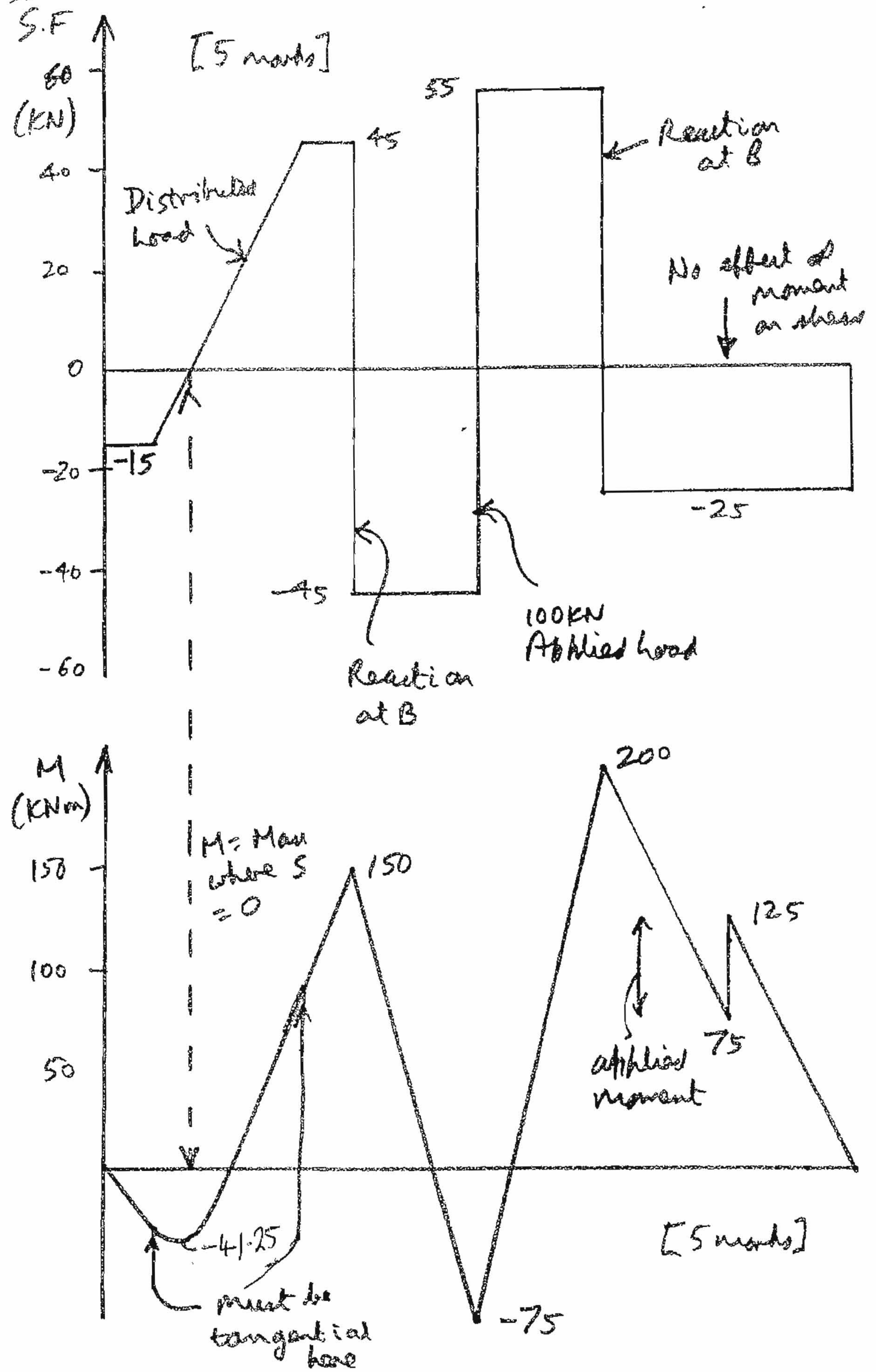
$$M_C = 25 \cdot 10 - 50 = \underline{\underline{200 \text{ KN m}}}$$

[5 marks]

To draw B.M & S.F diagrams, take free body cuts at appropriate places.

Common errors. Making it far too complicated. Not taking free body cuts carefully when constructing M + S diagrams. Question is not diffiuclt but many students tried to do it "by inspection" which does not work!

Q5


S.F
(KN)
[5 marks]
60
40
20
0
-20
-40
-60
-15
45
55
-45
-25
Distributed load
Reaction at B
No effect of moment on shear
Reaction at B
100KN Applied load
M
(KNm)
150
100
50
M = Max where S = 0
150
200
125
75
applied moment
-41.25
-75
must be tangential here
[5 marks]