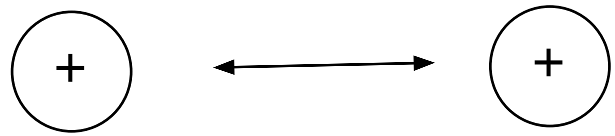
B and H

- *The truth at last?*

- *Archie Campbell*

Contents

Why do we Use Fields?



Force= $q_1 q_2 / 4\pi\varepsilon_o r^2$

The starting point is the law of force between charges

Define a field $E = q/4\pi\varepsilon_o r^2$

Potential $\varphi = q/4\pi\varepsilon_o r$

$E = -\text{grad}(\varphi)$

Force on q from many charges=qE

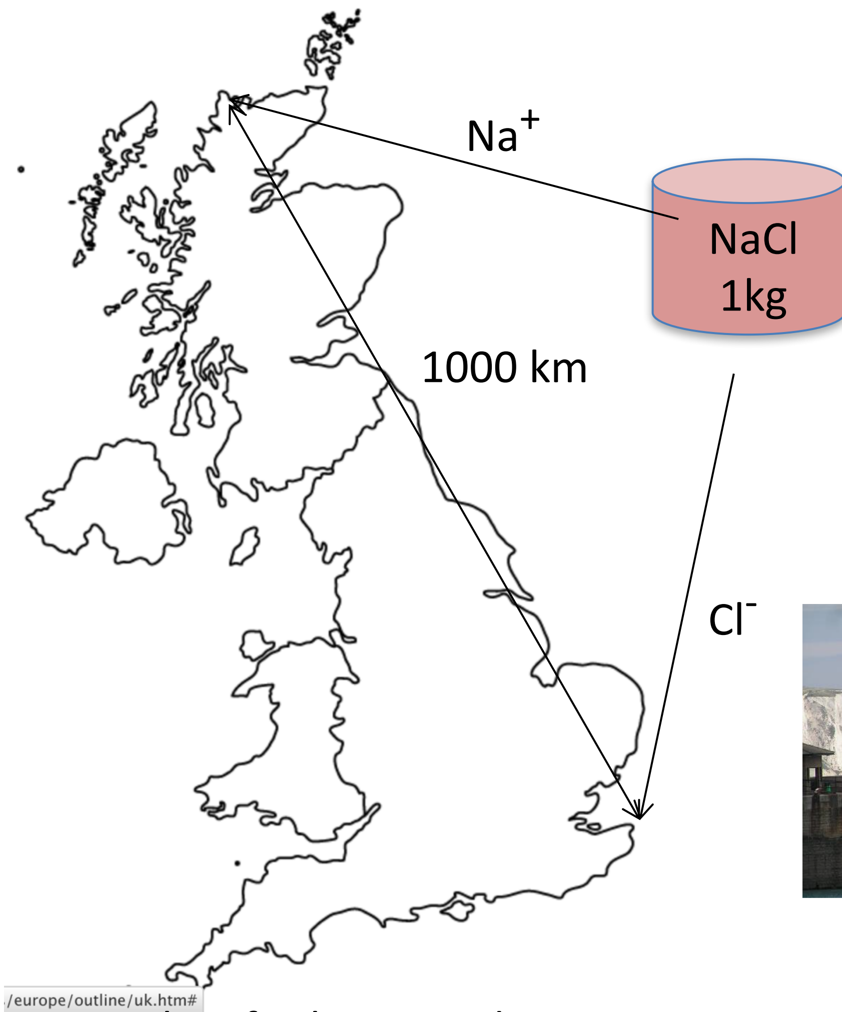
where E is the sum of the fields due to all other charges.

(this only works because free space is linear).

The field is an artificial concept which simplifies the calculation of the forces on charges. One value of the field can describe the force from many different arrangements of charge.

However like many such introductions it has gained a physical meaning in subsequent developments.

How strong are electrostatic forces?

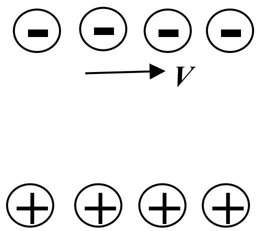


We separate 1kg of salt into sodium ions at Cape Wrath and Chlorine ions in Dover. Could we detect the force between them?

Yes

It is 28,000 million tonnes. Electrostatic forces are very strong. They hold us, and the universe, together on an atomic scale.

On the other hand magnetic Forces are very weak.



Consider two long lines of charge, density ρ,r apart one is moving with velocity v. The electrostatic force between them is $\rho^2/2\pi\varepsilon_o r$.

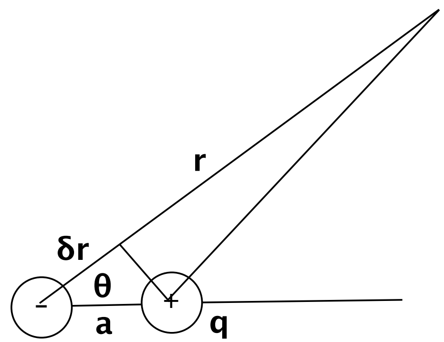
However since one is moving there is a small relativistic correction by a factor $1- v^2/c^2$.

This correction is $(\rho^2/2\pi\varepsilon_o r)v^2\mu_o\varepsilon_o = (\rho v)^2\mu_o/2\pi r = i^2\mu_o/2\pi r$

This is the magnetic force between the two currents. Since all our systems are electrically neutral we do not see the electrostatic force, only the magnetic one.

Since the drift velocity of the electrons in a mains wire carrying 13A is about 1 micron/sec this correction is about 1 part in 10^{24} and yet this is what we use to drive Eurostar trains, rather than electrostatic forces.

Single charges are rare and dissipate quickly and monopoles are even rarer. The fields we measure must be approximated by dipoles.



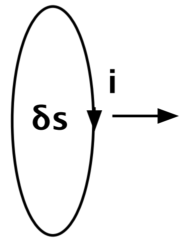
Dipole potential

$$\varphi = -q/4\pi\varepsilon_o r + q/4\pi\varepsilon_o(r-\delta r)$$
$$= q\,\delta r\ \cos(\theta)/4\pi\varepsilon_o r^2$$
$$= p\cos(\theta)/4\pi\varepsilon_o r^2$$

Where p=<u>qa</u> the dipole moment.

$$E = -\mathrm{grad}(\varphi)$$

Forces on Electrical Currents



It is found that the the forces between small current loops are identical to those between electrostatic dipoles if we define the dipole moment as

$m\delta = i.\delta S$

Where i is the current and δS the area

We define a magnetostatic potential ψ as

$\Psi = m \cos(\theta)/4\pi r^2$

Then define a field B as

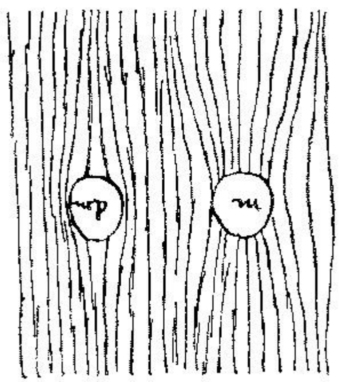
$B = \mu_o$ grad ψ and the force on a current I is then = B.I/length

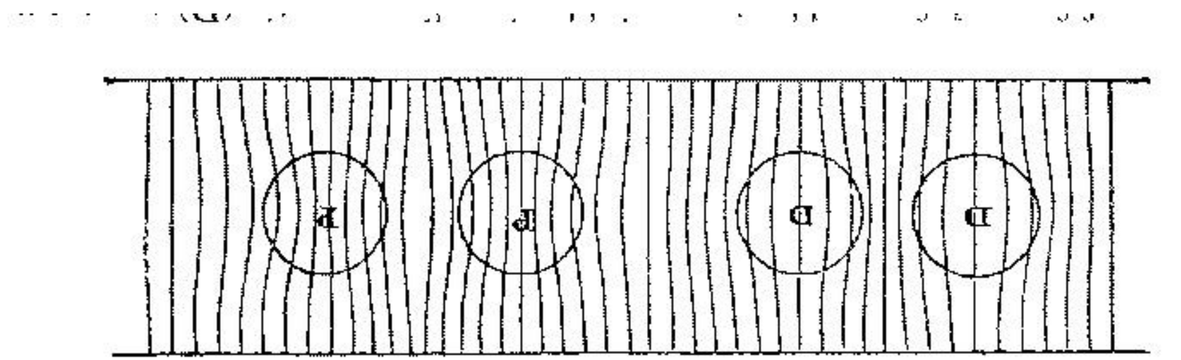
The μ_o appears rather unnaturally in the development for historical reasons , but this just so that we get the right units in Maxwell's equations at the end. There is only one field which can be called either B or H. They differ only in a factor μ_o according to whether we put them in Tesla or Amps/m

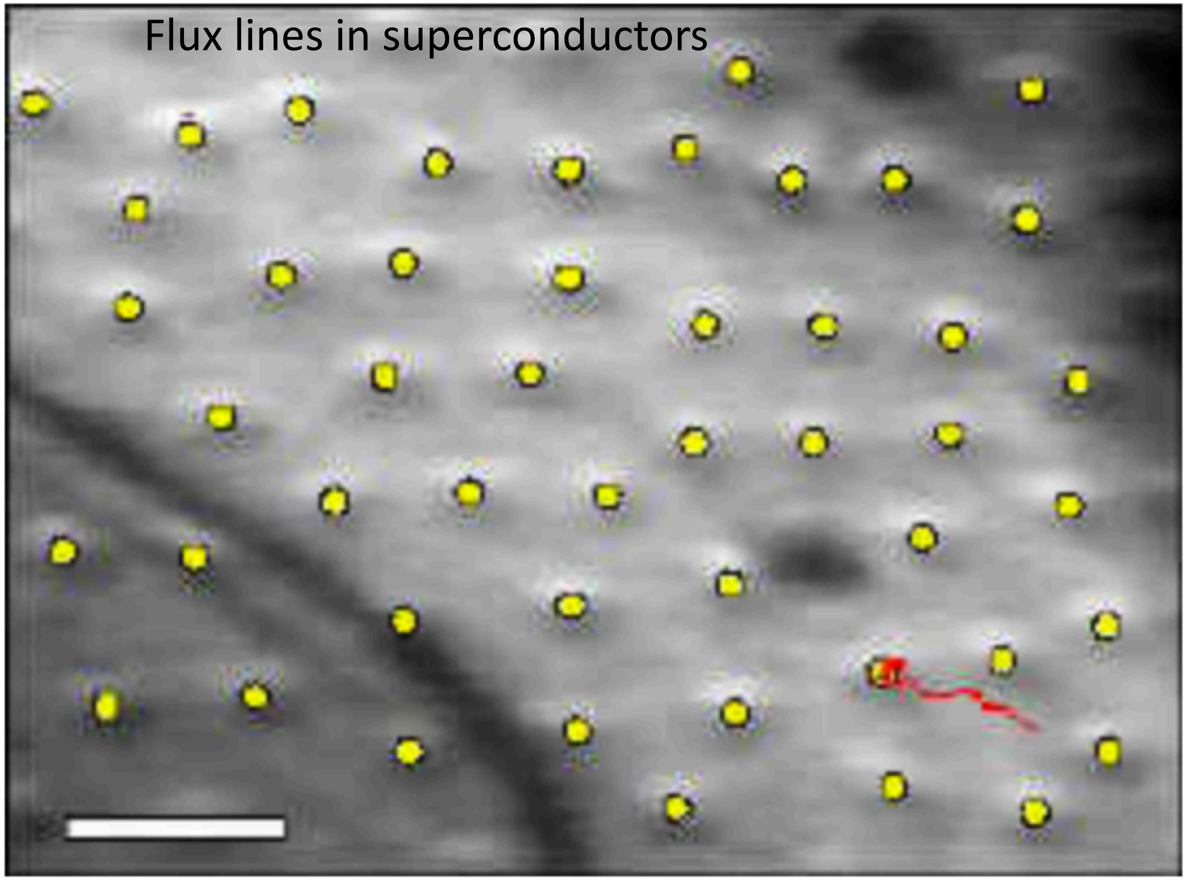
Lines of Force

Since the dipole field is based on the inverse square law we can apply Gauss's theorem and draw continuous lines of force.

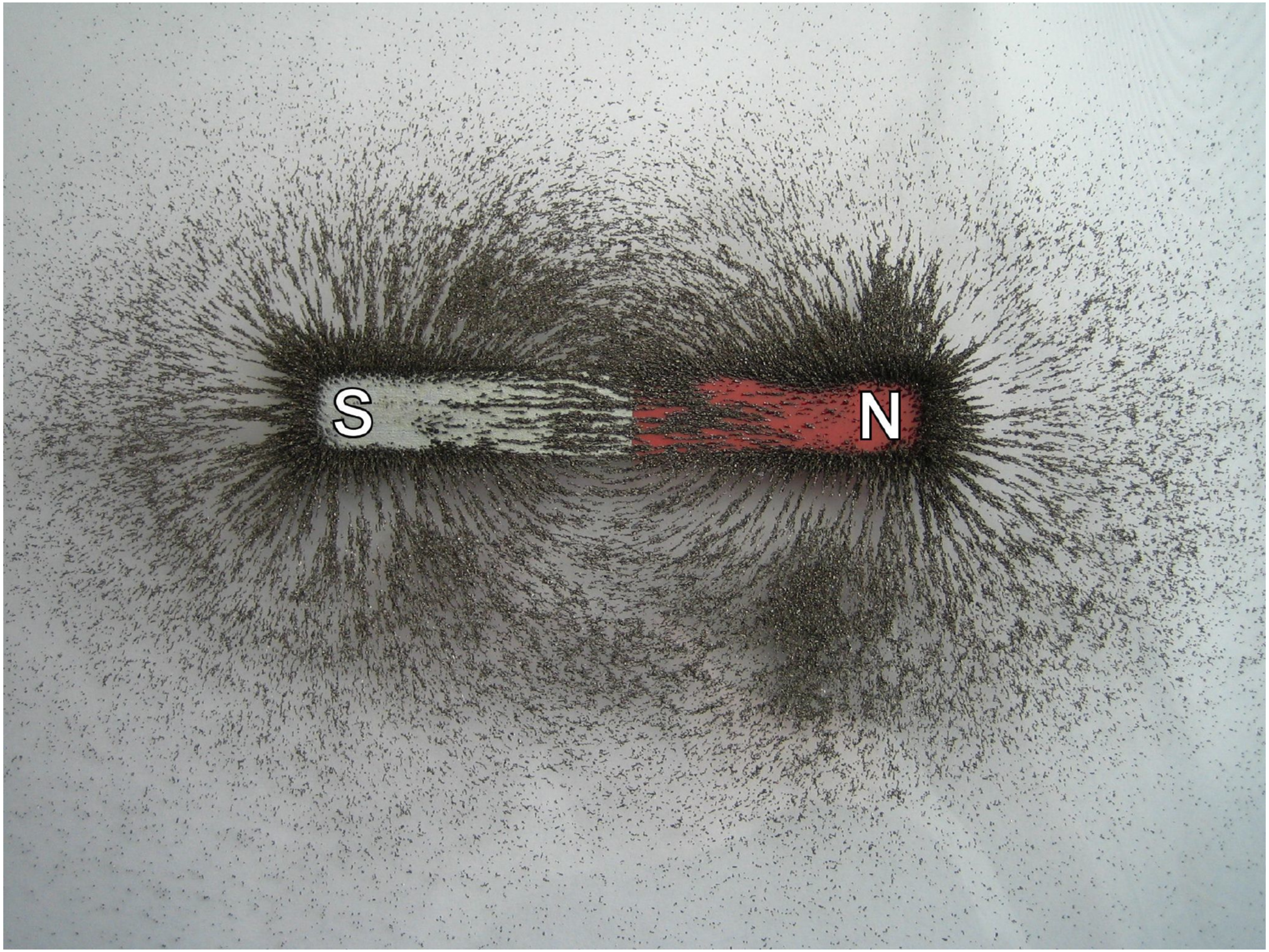
Faraday's notes





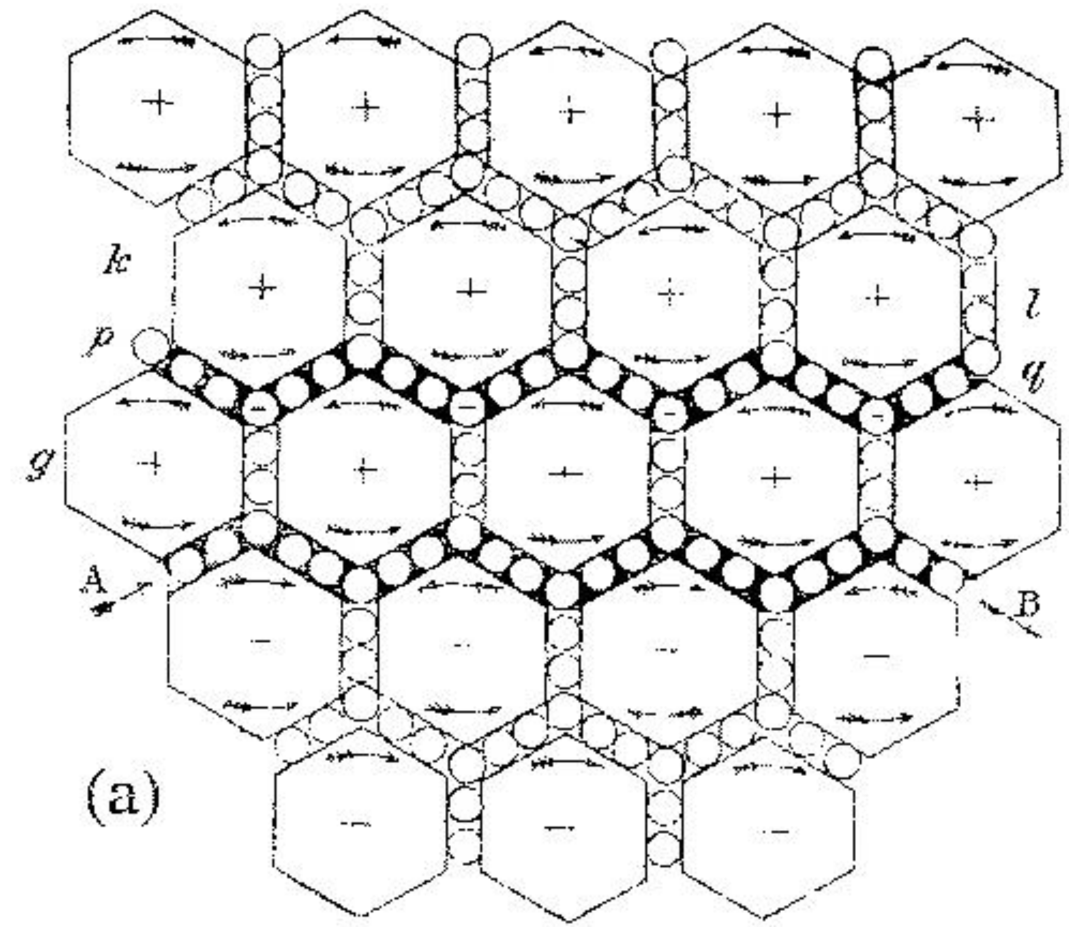


Phys. Rev. Lett. 80, 2693



Although a good illustration this is surprisingly difficult to explain

Maxwell's Picture of the magnetic field used 'vortices'



Field Equations for B in Free Space

From these definitions a little algebra shows that

$$\oint B.dS = 0$$ Gauss's theorem

and

$$\oint B.dl = \mu_o I$$ Ampere's circuital theorem

These are only useful in simple geometries. Integral equations are very difficult to solve.

By applying these equations to small loops Maxwell got local relations in the form of differential equations.

Div(B)=0 and Curl(B)=μ_oJ

Magnetic fields in materials, (not superconductors)

Magnetic materials are full of magnetic dipoles on an atomic scale so we need to average.

We define B in a material as the average of the local B on an atomic scale, b.

Averaging Gauss's theorem div(b)=0 gives div(B)=0.

Averaging curl(b)=μ_oJ is more problematical and will be done in two ways.

The first shows how a magnetic array can be replaced by equivalent currents. This is a very useful construction.

We first define the second fundamental field, **the magnetisation**.

Magnetisation

Magnetic materials consist of spinning electrons and orbiting electrons each of which behaves as a dipole at distances large compared with the size of an atom. The local magnetisation ***M*** is defined as the sum of the magnetic moments over a small volume, divided by the volume, the mean magnetic moment per unit volume

$$M = \sum m / \delta V$$

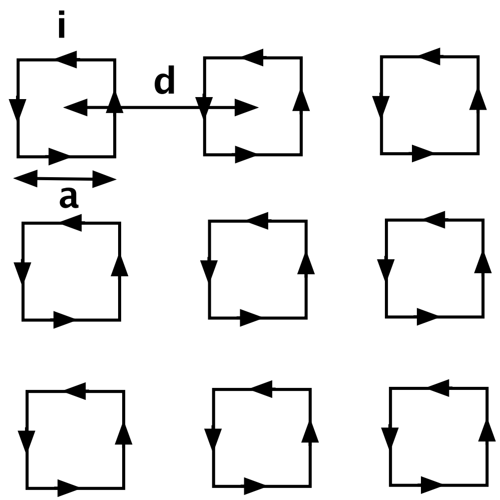
(compare charge density $\rho = \sum q / \delta V$)

The distance over which we we average is normally large compared with the microstructure but small compared with the sample size.

The vector fields **B** and **M** are the fundamental fields in materials.

Equivalent Currents

Consider an array of square dipoles of side *a* space d apart carrying a current *i* in layers *d* apart normal to the paper



A dipole array

The magnetisation $M=ia^2/d^3$ A/m.
This is an array of solenoids with current density *id* A/m.
The local *b* in each is $\mu_o i/d$ and zero outside so the average is $B=(\mu_o i/d)(a/d^2)=\mu_o M$

This is the standard result for *B* in a long thin magnet.
This illustrates that at least for a simple array B is the average of the free space field

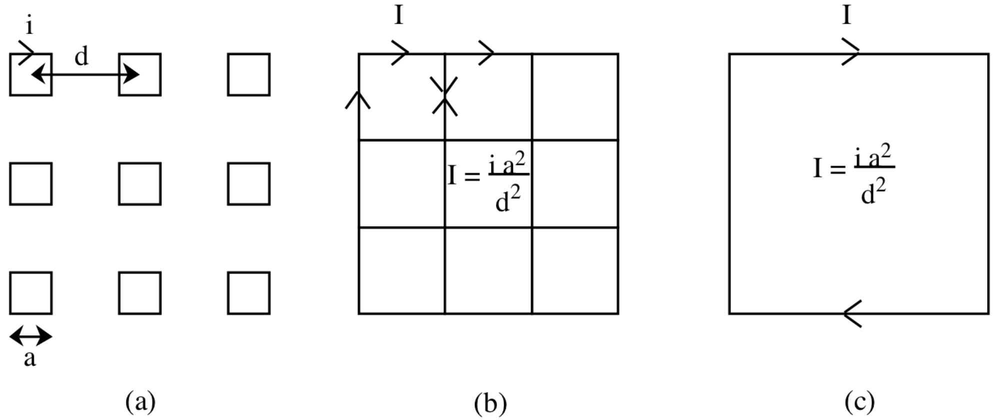


Fig.1. (a) A dipole array. (b) Larger dipoles with the same magnetisation. (c) The Equivalent surface Current.

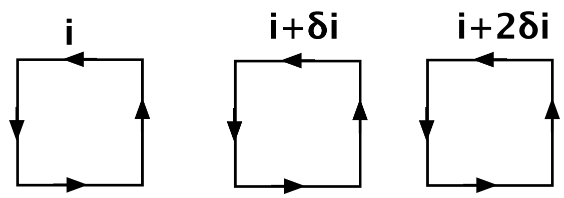
Now multiply the edge length by *d/a* and divide the current by $(d/a)^2$.

This keeps the magnetisation the same but now the internal currents cancel and can be removed leaving a surface current $I=ia^2/d^2$ in each layer or $ia^2/d^3=M$ Amp/m on the whole surface.
This also has the same average B.

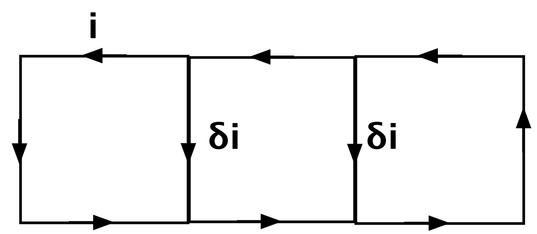
We can get the same B every where if we replace the magnetisation by an equivalent surface current M A/m.
In general it is the parallel component of M

(1 Amp/m=1/μ_o Tesla)

This was for a uniform magnetisation
If M is changing, with x for example



Increase size while keeping M the same



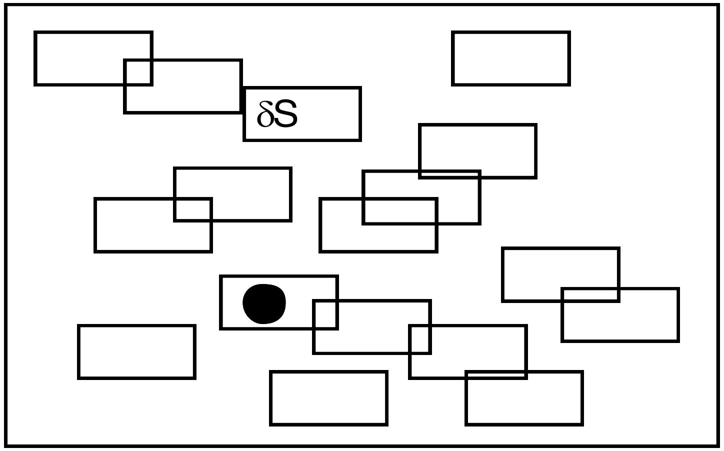
$$J_y = \delta i\left(d^2 / a^2\right) / a^2 = -dM_z / dx$$

This is one component of the general equation curl(M)=J
Note that this J is only magnetisation currents, transport currents are separate and can be added on.

Thus a uniform magnetisation can be replaced by a surface current.
A non-uniform magnetisation needs a bulk current density as well.

These currents in free space give the same flux density B as in a magnetised the material.

A More General Derivation



The figure shows a random array of dipoles in a thickness δl projected onto the xy plane.
A line perpendicular to this intersects the plane at the black dot.

Ampere's theorem $\oint b.dl=\mu_o I$
Averaging $\oint B.dl=\mu_o \bar{I}$
where $\bar{I}$ is the mean current intersected

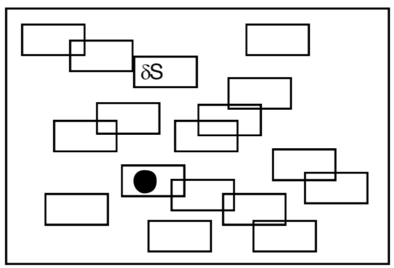
We need to work out the mean current intersected by the line of length δl.
The probability of intersecting a particular dipole is $\delta S/A$ where δS is its area and A the area of the array. The mean current contributed is $i\ \delta S/A$ where i is the current in the dipole.
The total of the array is $\sum i\ \delta S/A=M\delta l$ where M is the magnetisation.
If we add the magnetisation currents $M\delta l$ and a transport current I in Amperes relation we get

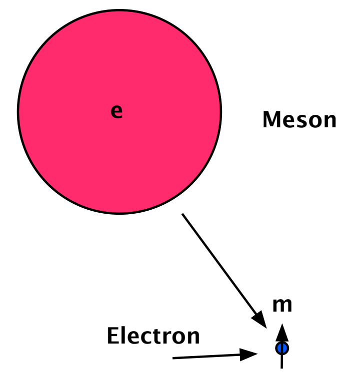
$$\oint \boldsymbol{B.dl}=\mu_o\oint \boldsymbol{M.dl}+\mu_o \boldsymbol{I}$$

This is really all we need. It is Ampere's equation in a magnetic material.
Combined with the material property $M(B)$ the field equations can be solved.

Experimental Justification

There is a small number of experiments which show that **B** is the average of the local field. We need a charged particle which is going so fast that it is not deflected by atomic charges. Rasetti (1) used cosmic ray mesons. Hughes (2) used neutrons. Both are much bigger than electrons. To sample the mean field the complete meson must pass through the the centre of the electron!





1).F.Rasetti, Phys.Rev. **66 ,**1 1944**.**
2) D.J.Hughes, Pile Neutron Research, Addison Wesley, 1953 sections 11-4 and 10-6.
3**)** G.H.Wannier, Phys.Rev., **72**, 304 1947

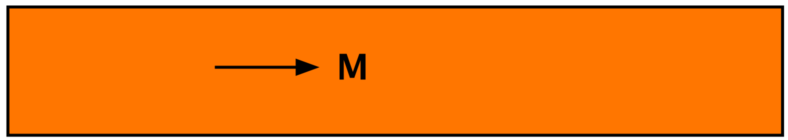
Apparently it does, the full theory needs the Dirac electron theory (3).

More recently muon spin decay has been used to map local fields in superconductors. This works on the scale of the penetration depth, but not for B in iron. The muons prefer to congregate at the sides of atoms where the field is more like H than B, but not equal to either.

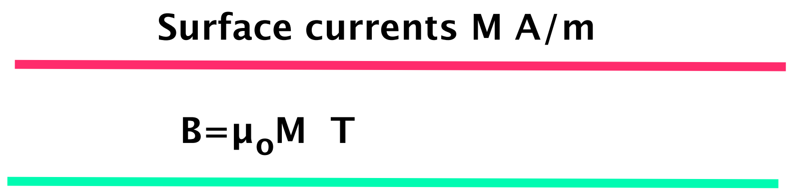
NdFeB makes things very easy. M is constant

(In data sheets it is the ‘Remanence’/μ_o=1-1.5T)

Here are two examples

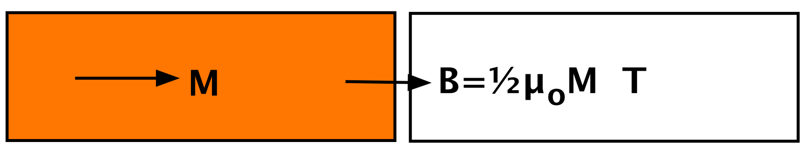


Long thin magnet



Equivalent currents

Long solenoid formula



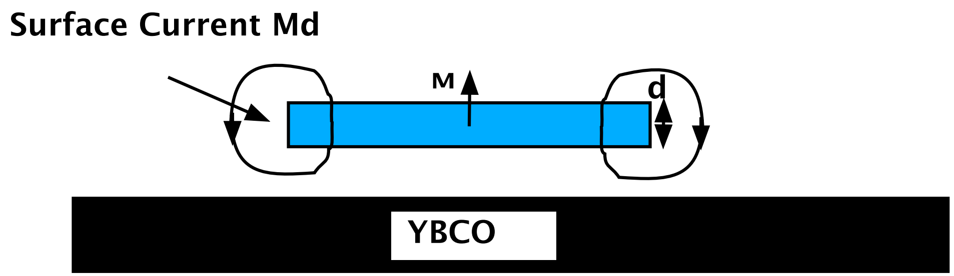
Two half magnets must add up to one whole one

Normal surface field is uniform and half central field

(However the radial field is maximum at the edges and there is a singularity in the magnitude of B)

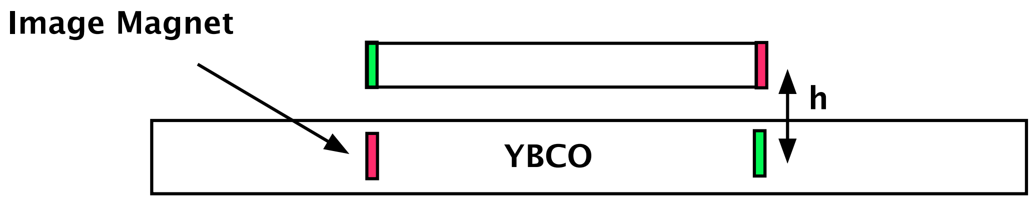
Levitation

Disc Magnet Radius R



$$Force=2\pi R(Md)^2/2\pi\mu_o h$$

$$Pressure=(Md)^2/2\pi\mu_o hR$$



The magnet is repelled by its image in the superconductor.
The Pressure goes to zero for large diameter. We need to subdivide the magnets.

The Field H

Ampere in material $\oint \mathbf{B.dl} = \mu_o \oint \mathbf{M.dl} + \mu_o \mathbf{I}$

That is the all the electromagnetism we need, and we have not needed an H or a permeability, μ.

However it is convenient to define a new vector H as

$$\mathbf{H = B/\mu_o - M.}$$

Then substituting for M

$$\oint \mathbf{H.dl = I}$$

What Maxwell did was to apply this to a small loop and turn a definite integral into a differential equation

curl(H)=J

It is usually more convenient to use B(H)rather than M(B) on solving equations but the two are equivalent

In practical superconductors there are no magnetic dipoles so M=0 and $B=\mu_o H$.
(but see Evetts theory)

What is Magnetisation?

The term has several meanings, which must be kept clear.

So far it is a local vector, the average density of magnetic dipoles in the same way that charge density is the density of single charges. It is only in this sense that $B=\mu_o(H+M)$, but remember this is the definition of H, not B. It allows us to separate the currents due to dipoles from those due to transport currents. It can be applied to superconductors using the Evetts theory, not discussed here, but this is not of practical significance.

We can also define the total magnetic moment of a body

$$\mathbf{M_o=½\int r x j\ dv.}$$

This can only be defined for an isolated body with no currents flowing in or out. However it includes eddy currents and currents in superconductors. It determines the distant magnetic field from any body and is what a SQUID magnetometer measures. If divided by the volume, or for a body of unit volume, it is also called magnetisation, but don't confuse the two definitions They can be quite different.

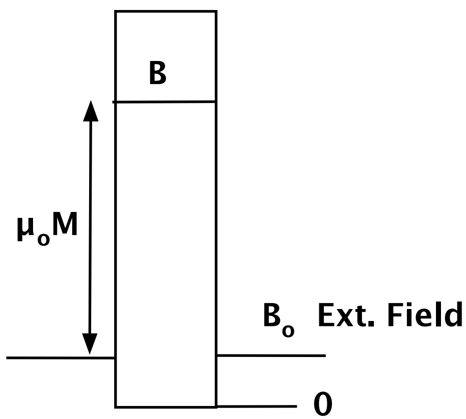
For currents in a plane we can integrate the current loops to find M_o.

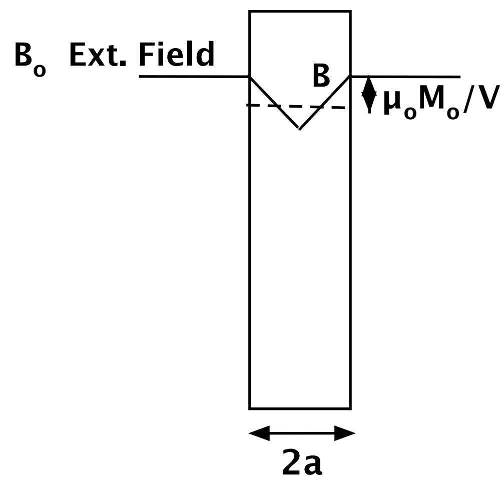
Some Special cases

If there are no macroscopic transport currents

Total moment $M_o = \int M \, dV$

If we have a long thin cylinder in a parallel field B_o then M is the difference between the external field and the mean flux density.





Long thin ferromagnet.
No currents.
$M_o = MV$
$H = H_o = B_o/\mu_o$
$B = \mu_o H + \mu_o M$

Long thin cylindrical superconductor.
Pinned transport currents.
$M = 0$
$M_o/V = 1/3\, J_c\, a$
$B = \mu_o H$
$dB/dr = \mu_o dH/dr = \mu_o J_c$

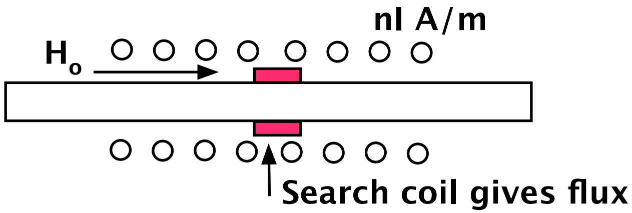
The Applied Field H_o

The applied field can only be defined for an isolated body. It is the field in the space left by the body if it is removed, and all currents, including all magnetisation currents, except those of the body, are kept constant. It must be done this way round, rather than by adding the body to an existing field, as a magnet near a permeable material experiences an applied field due to its image, which must be preserved when the body is removed. (However in simple geometry the applied field is also the field a long distance from a body).

Many problems involving forces and energies are much more easily tackled using the applied field $\boldsymbol{H_o}$ and the total magnetic moment of a body, $\boldsymbol{M_o}$, rather than the local values $\boldsymbol{H}$ and $\boldsymbol{M}$ or $\boldsymbol{B}$ which must be integrated over all space (see the section on energies and forces). This is because we can express the work done on a system in terms of $\boldsymbol{M_o}$ and $\boldsymbol{H_o}$, whereas to find the energies we need to integrate fields over all space which is algebraically intractable.

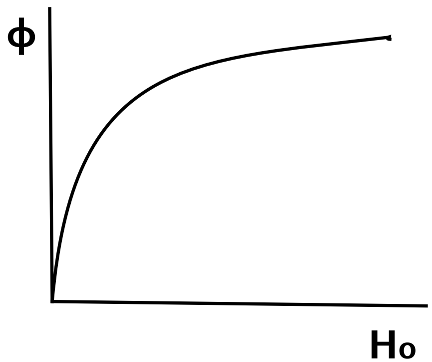
In some geometries the external and local values are the same, but it is nevertheless extremely important to make clear which are being used.

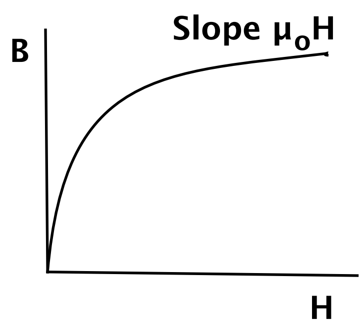
Material Properties , B(H) curve

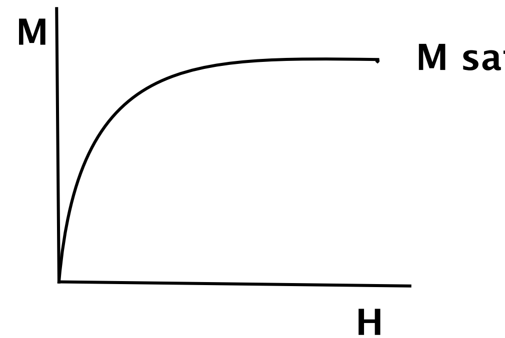


Apply an external field $H_o=B_o/\mu_o=NI$ and measure flux with a search coil. We need a very long thin uniform sample, or better a toroid.

Since there is no transport current on the surface H in material $=H_o$







Experiment

Relabel Axes
$B=\varphi/\text{Area}$
$H=H_o$

General Material Property
, B(H) curve.
This true in all circumstances

$M=B/\mu_o-H$
M sat about 3 Bohr Magnetons/ atom

Demagnetising Effects

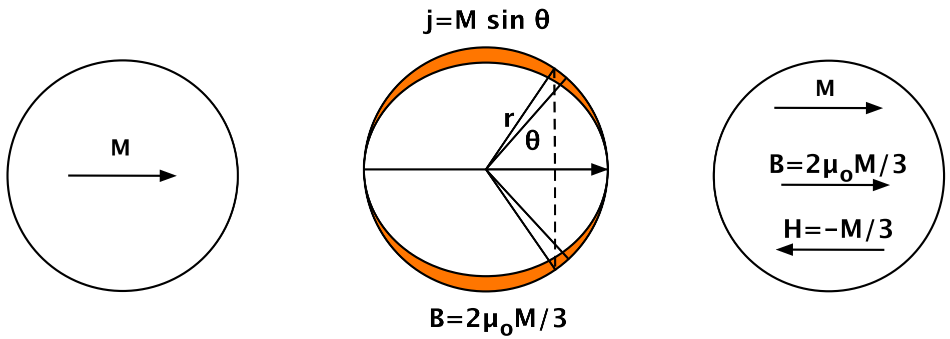
Most undergraduate courses start with linear materials and go on to non-linear and hysteretic materials as a special case. This is a big mistake. Once undergraduates get the idea that $B=\mu\mu_o H$ is a fundamental equation they will never understand the subject. The reason is that the application of an external field leads to a magnetisation, which then adds a field to the applied one. There is a large amount of feedback and, as always, this leads to counterintuitive results.

In this respect electromagnetism differs from mechanics, elasticity or thermodynamics.

The effects may be described as 'Demagnetising Effects'.

Demagnetising factors

Material Free space Currents Fields in Material

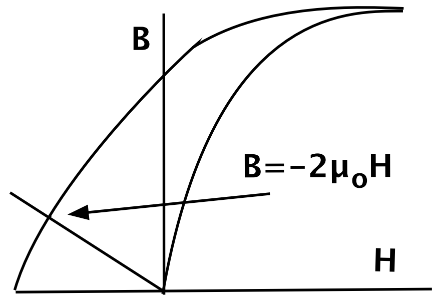


Start with a uniformly magnetised sphere, magnetisation M.

Replace with equivalent surface currents M sinθ/length.
Find B at the centre from Biot Savart Law .(In fact B is uniform)
$B=2\mu_o M/3$ (same direction as M)
Then $H=B/\mu_o-M=-M/3$.
The 1/3 is the demagnetising factor, n, for a sphere.

We see $B=-2\mu_o H$
If we draw a line of this slope on the B-H curve we find the effect of demagnetisation

The general definition of n is from $H=-nM+H_o$



This allows us to determine the magnetisation of a permanent magnet from the B-H curve.

From the equivalent currents above, for a magnetised sphere $B=-2\mu_o H$. Where the line of slope $-2\mu_o H$ intersects the B-H curve we get the B and hence M allowing for demagnetisation.

This limited the magnetisation of permanent magnets before the invention of NdFeB.

Previously permanent magnets had to be long and thin, or inserted in a long high permeability magnetic circuit.

Linear Materials

In some materials the orientation of dipoles is proportional to the field on them so that all fields are linearly related.

We write $M=\chi H$ where χ is defined as the susceptibility, a material parameter.

Then $B=\mu_r\mu_o H$ where $\mu_r=1+\chi$ and is called the relative permeability.

Materials are either ferromagnetic, in which case μ_r and χ are large and nearly equal (>300),

or they are not , in which case χ is small $<10^{-5}$, $\mu_r \sim 1$ and $B \sim \mu_o H$

In ferromagnetic materials demagnetising effects are extremely important.

Consider an ellipsoid of demagnetising factor n in a uniform external field H_o

Then $H=H_o-nM$ and $M=\chi H$.

$M=\chi H_o/(1+n\chi)$.

For non ferromagnetic $M=\chi H_o$

For a ferromagnet $M=H_o/n$.

e.g. for a sphere $M=3H_o$ independent of permeability

Only for very long thin samples parallel to the field does M become large.

Magnetic forces on ferromagnetic bodies are usually independent of the permeability

The magnetisation is of the same order as the applied field

What is H?

It is easier to say what it is not. Here are some ‘explanations’ which are at best of very limited application

1) H is the ‘external field’.
Only in a long thin sample parallel to a uniform external field with no currents flowing is H in the material equal to H_o.

2) The ‘H’ produces a ‘B’ or an ‘M’.
This is a meaningless statement. It is like saying a ‘stress’ produces a ‘strain’ or a ‘voltage’ produces a ‘current’ when frequently it is the other way around. In all these cases it is an artefact of the experimental setup and depends on the rigidity or source impedance of the apparatus

3) H is the ‘stray’ field.
I had some difficulty working out what this means. I think it refers to a long thin permanent magnet where the field just outside the centre of the magnet is equal to H in the magnet. This is not a useful concept.

4) ‘H is the field due to transport currents without the effect of magnetisation’.
If this were true we could throw away all our finite element packages as the field from transport currents can be found from the Biot Savart law. Although cur(H)=J the solutions of this equation are multiple and we need more information.

5) H is the ‘internal field’ .
‘Internal field’ is not usually defined, but the field in a material varies widely on an atomic scale so this is a meaningless definition.

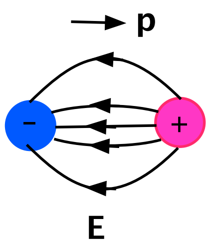
6) ‘The H field determines the force on currents and the B field the induced voltage’.
This probably had its origins in the nineteenth century when it was only gradually becoming clear that the same field in free space determined both the force on a current and the induced voltage. That there was no difference was finally proved by the theory of relativity.

Extending this idea to fields in a material is complete nonsense. It was largely based on the idea of defining fields by the forces on bodies in magnetic fluids which led to different macroscopic formulations by Sommerfeld and Connelly. This controversy is a pointless argument only of interest to historians of science.

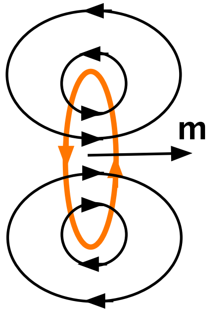
The forces on bodies in fluids are apparently simple but in fact are a complex combination of hydrostatic and electromagnetic forces. In solids they become tensors and are even more complicated and involve not only the fields but the rate of change of permeability with stress.

Atomic Scale Interpretations

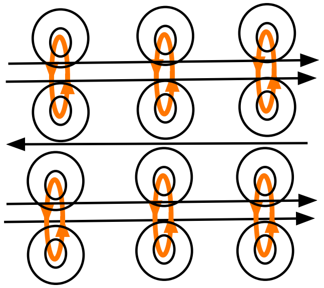
A major difference between electrostatic and magnetic dipoles is the direction of their mean field



Electric dipole.
E in opposite direction to p

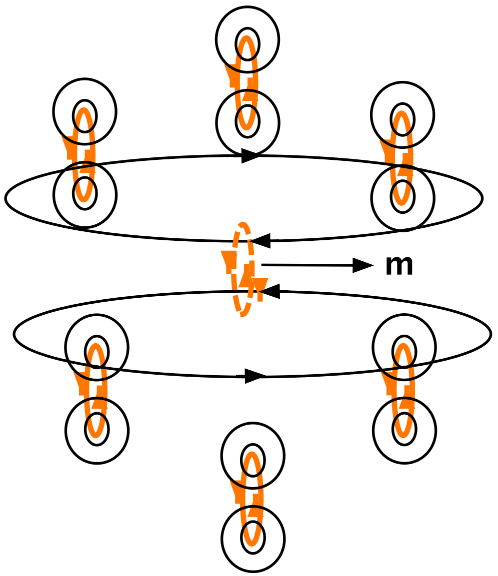


Magnetic Dipole
Mean field parallel to m



6) ‘H is the mean field along a line which does not intersect any dipoles’.
This true , but not particularly illuminating.

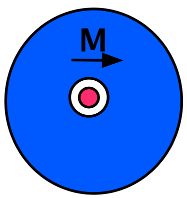
7) ‘H is the field on a dipole tending to demagnetise it’.
There is some truth in this, but it is far from the whole story.



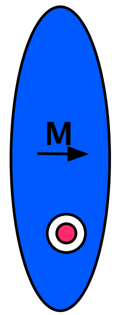
The field applied to the central dipole in this array (and to all the others in a large array) is in the opposite direction to m and so tends to demagnetise it.
However this neglects the field due to the surface current of the sample which is taken into account in the Lorentz theory.

Lorentz Theory

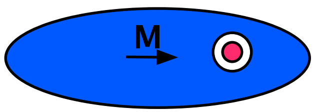
Lorentz showed that the field of dipoles within a sphere in symmetric crystals summed to zero. Hence the field on an atom is the field due to surface currents on the sphere round it , +1/3 M. To this must be added the field due to surface currents on the sample surface.



Here there is no field on the atom.
Fields due to the two surfaces are + and – M/3.
However H=-M/3



Here the field Demagnetises a dipole.
The sample surface currents Give a field<-M/3



Here the field reinforces the magnetisation although H is in the opposite direction to M.
The sample surface currents Give a field>-M/3

However in all cases H=-nM

Thermodynamics

The discovery of superconductors required a rethink of what we mean by H and M in a material.
The answer was provided independently in slightly different ways by Josephson and Evetts.

Brian Josephson considered materials in thermodynamic equilibrium and defined **H** as the gradient of the free energy w.r.t. to B. i.e. by δF=**H.dB**

Jan Evetts defined **H** in superconductors by H(B)= external field in equilibrium with B.
i.e H(B) is the Abrikosov curve and M the reversible Abrikosov magnetisation.
H can be regarded as the chemical potential of a flux line per unit length and since curl(H)=J the curl allows for the line tension.

This is a very beautiful piece of physics, but since M is so small it can usually be ignored in high κ materials.

Both treatments bring out the real nature of H which is to indicate the degree the system departs from thermodynamic equilibrium.

Summary

We define the field of a dipole in free space so that it gives the correct forces on currents.

With a bit of algebra this leads to Gauss, Ampere and Maxwell's equations in free space.

We define B as the average of the microscopic field and M as the dipole moment per unit volume.

We divide currents into magnetisation currents curl(M) and transport currents.

Then Ampere becomes $\oint \mathbf{B.dl=\mu_o \oint M.dl+\mu_o I}$

Define $\mathbf{H=B/\mu_o-M}$. Then $\mathbf{curl(H)=J}$

Always make sure to to distinguish between the local magnetisation M due to magnetic dipoles and the total magnetic moment M_o which will include eddy and pinned macroscopic currents.

Also distinguish between the H in the material and the external field H_o.