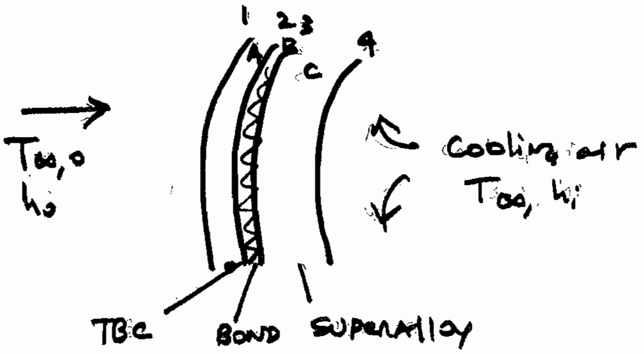
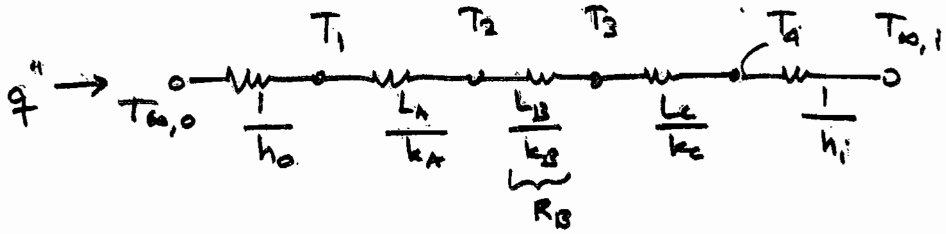
1



(a)



	L(m)	k(W/mK)
A	0.5×10^{-3}	1.3
B	$R_B = 10^{-4}$ (W/m²K)	
C	5×10^{-3}	25

$$q'' = \frac{T_{\infty,o} - T_{\infty,i}}{\sum R_i}$$

(I) No TBC, No BONDING AGENT

$$\sum R_{i,I} = \frac{1}{h_o} + \frac{L_C}{k_C} + \frac{1}{h_i} =$$

$$\sum R_{i,I} = \frac{1}{1000\ \mathrm{W/m^2K}} + \frac{5\times10^{-3}\ \mathrm{m}}{25\ \mathrm{W/mK}} + \frac{1}{500\ \mathrm{W/m^2K}} = 3.2\times10^{-3}\ \frac{\mathrm{m^2K}}{\mathrm{W}}$$

(II) w/ TBC + bonding agent:

$$\sum R_{i,II} = \sum R_{i,I} + \frac{L_A}{k_A} + R_B = 3.2\times10^{-3}\ \frac{\mathrm{m^2K}}{\mathrm{W}} + \frac{0.5\times10^{-3}\ \mathrm{m}}{1.3\ \frac{\mathrm{W}}{\mathrm{mK}}} + 10^{-4}\ \frac{\mathrm{W}}{\mathrm{m^2K}}$$

$$\sum R_{i,II} = 3.68\times10^{-3}\ \frac{\mathrm{m^2K}}{\mathrm{W}}$$

(b)
$$q_I = \frac{T_{\infty,o} - T_{\infty,i}}{R_I} = \frac{1700 - 400\ \mathrm{K}}{3.2\times10^{-3}\ \frac{\mathrm{m^2K}}{\mathrm{W}}} = 4.06\times10^{5}\ \mathrm{W/m^2}$$

$$q_{II} = \frac{T_{\infty,o} - T_{\infty,i}}{R_{II}} = \frac{1700-400\ \mathrm{K}}{3.68\times10^{-3}\ \frac{\mathrm{m^2K}}{\mathrm{W}}} = 3.53\times10^{5}\ \mathrm{W/m^2}$$

TEMPERATURE DISTRIBUTION:

$$T_i = T_{i-1} - q'' R_i$$

(b)

CASE I :

$T_3 \quad T_{\infty,o} - q''_I(1/h_o) = 1700 - \dfrac{(4.06\times10^5\,W/m^2)}{1000\,W/m^2K} = \boxed{1294\,K}$

$T_4 \quad T_1 - q''_I\left(\dfrac{L_c}{k_c}\right) = 1294 - \dfrac{4.06\times10^5\,W/m^2}{(25\,W/mK)/(5\times10^{-3}\,m)} = 1213\,K$

$T_4 = \quad T_{\infty,i} + q''_I\left(\dfrac{1}{h_i}\right) = 400 + \dfrac{4.06\times10^5\,W/m^2}{500\,W/m^2K} = 1213\,K$

ok

CASE II :

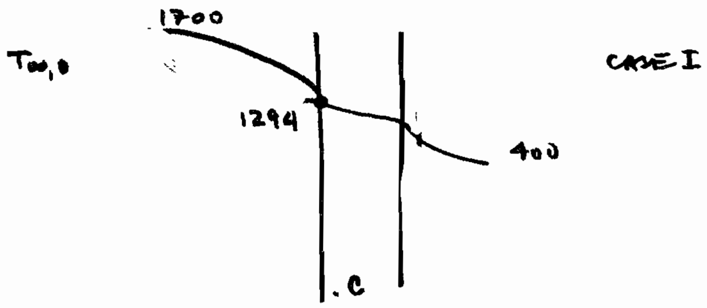
$T_1 = T_{\infty,o} - q''_{II}\left(\dfrac{1}{h_o}\right) = 1700 - \dfrac{3.53\times10^5\,W/m^2}{1000\,W/m^2K} = 1347\,K$

$T_2 = T_1 - q''_{II}\left(\dfrac{L_A}{k_A}\right) = 1347 - \dfrac{3.53\times10^5\,W/m^2}{\left(\dfrac{1.3\,W/m\,K}{0.5\times10^{-3}\,m}\right)} = 1211\,K$

$T_4 = T_{\infty,i} + q''_{II}\dfrac{1}{h_i} = 400 + \dfrac{3.53\times10^5\,W/m^2}{500\,W/m^2K} = 1106\,K$

$T_3 = T_4 + q''_{II}\dfrac{L_c}{k_c} = 1106 + \dfrac{3.53\times10^5\,W/m^2(5\times10^{-3}\,m)}{(25\,W/mK)} = \boxed{1176\,K}$

TEMPERATURE PROFILES.

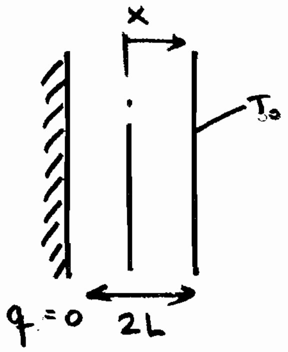


(c) THEREFORE, THE USE OF TBC REDUCES THE MAXIMUM TEMPERATURE OF THE INCONEL FROM 1294 K DOWN TO 1176 K, TO BRING IT WITHIN THE ALLOWABLE MATERIAL LIMITS.

THE THICKNESS OF THE ~~[illegible]~~ TBC IS LIMITED BY THE QUALITY OF THE BOND. THICK CERAMIC BARRIERS CAN FLAKE AND INTRODUCE SMALL PARTICLES INTO THE HIGH SPEED STREAM, WHICH CAN THEN ERODE OTHER BLADES.

2 (a)

CASE 1



GENERAL EQUATION FOR SS WITH HEAT GENERATION AND UNIFORM k:

$$k\nabla^2 T + \dot{q} = 0$$

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0$$

INTEGRATE TWICE:

$$\frac{dT}{dx} = -\frac{\dot{q}}{k}x + c_1$$

$$T(x) = -\frac{\dot{q}}{k}\frac{x^2}{2} + c_1 x + c_2$$

B.C.: $$\left.\frac{dT}{dx}\right|_{x=-L} = -\frac{\dot{q}}{k}(-L) + c_1 = 0 \qquad \text{(NO HEAT FLUX)}$$

$$c_1 = -\dot{q}\frac{L}{k}$$

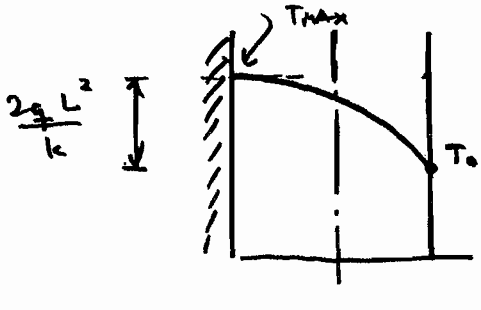
$$T(L) = -\frac{\dot{q}}{k}\frac{(L)^2}{2} + \left(-\dot{q}\frac{L}{k}\right)(L) + c_2 = T_0$$

$$c_2 = T_0 + \frac{3}{2}\frac{\dot{q}}{k}L^2$$

$$T(x) = -\frac{\dot{q}}{k}\frac{x^2}{2} + \left(-\frac{\dot{q}L}{k}\right)x + T_0 + \frac{3}{2}\frac{\dot{q}}{k}L^2$$

$$= \frac{\dot{q}L^2}{k}\left(-\frac{x^2}{2L^2} - \frac{x}{L} + \frac{3}{2}\right) + T_0$$

$$\frac{dT}{dx} = \frac{\dot{q}L^2}{k}\left(-\frac{2x}{2L^2} - \frac{1}{L}\right) = 0 \qquad x = -L \qquad \text{max. T}$$



$$T(-L) = \frac{\dot{q}L^2}{k}\left(-\frac{L^2}{2L^2} + \frac{L}{L} + \frac{3}{2}\right) + T_0$$

$$= 2\frac{\dot{q}L^2}{k} + T_0$$

$$= \frac{2(5\times10^6\,W/m^3)(0.020m)^2}{(50\,W/m\,K)} + 50^\circ C$$

$$T(-L) = 130^\circ C$$

(b) CASE 2

THE GENERAL equation FOR EACH HALF SECTION IS THE SAME AS IN CASE 1. THE BOUNDARY CONDITIONS ARE DIFFERENT, HOWEVER.

$$T_A(x) = -\frac{\dot{q}x^2}{2k} + C_{A1}x + C_{A2}$$

$$T_B(x) = -\frac{\dot{q}x^2}{2k} + C_{B1}x + C_{B2}$$

$$\left.\frac{dT_A}{dx}\right|_{x=-L} = 0 \implies -\frac{2\dot{q}(\cancel{x}-L)}{2k} + C_{A1} = 0 \implies C_{A1} = -\frac{\dot{q}L}{k} \qquad (1)$$

$$T_B(L) = T_0 \implies -\frac{\dot{q}}{2k}L^2 + C_{B1}L + C_{B2} = T_0 \qquad (2)$$

MATCHING CONDITIONS @ $x=0$

$$q_A'' = q_B'' = \frac{T_A(0) - T_B(0)}{R_t} \qquad (3),(4)$$

$$q_A'' = -k\left.\frac{dT_A}{dx}\right|_{x=0} = -kC_{A1}$$

$$q_B'' = -k\left.\frac{dT_B}{dx}\right|_{x=0} = -kC_{B1}$$

$$\therefore \quad C_{A1} = C_{B2} = \frac{C_{A2} - C_{B2}}{-kR_t} = -\frac{\dot{q}L}{k} = C_{A1}$$

FROM (2): $$T_0 = -\frac{\dot{q}L^2}{2k} + \left(-\frac{\dot{q}L}{k}\right)L + C_{B2}$$

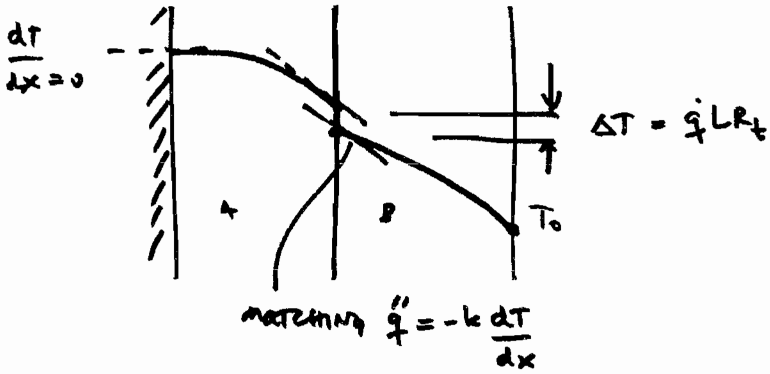
$$C_{B2} = T_0 + \frac{3}{2}\frac{\dot{q}L^2}{k}$$

$$C_{A2} = C_{B2} + (\dot{q}L)R_t = T_0 + \frac{3}{2}\frac{\dot{q}L^2}{k} + \dot{q}LR_t$$

$$T_A(x) = \underbrace{-\frac{\dot{q}x^2}{2k} + \left(-\frac{\dot{q}L}{k}\right)x + T_0 + \frac{3}{2}\frac{\dot{q}L^2}{k}}_{T(x),\ \text{CASE 1}} + \dot{q}LR_t$$

$$T_B(x) = \underbrace{-\frac{\dot{q}x^2}{2k} + \left(-\frac{\dot{q}L}{k}\right)x + T_0 + \frac{3}{2}\frac{\dot{q}L^2}{k}}_{T(x),\ \text{CASE 1}}$$

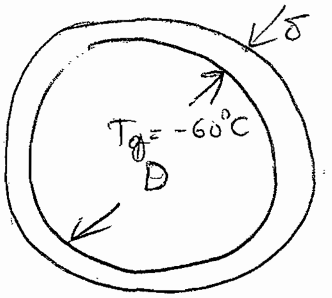
TEMPERATURE PROFILE :



(c) $T_A(0) - T_B(0) = \dot{q} L R_t = \left(5 \times 10^6 \frac{W}{m^3}\right)(0.020 m)\left(0.0005 \frac{m^2 K}{W}\right) = 50 K$

THEREFORE, THE PROFILES ARE THE SAME, BUT THE TEMPERATURES OF SIDE A ARE SHIFTED UP BY 50K, IE $T_A = 130 + 50 = 180°C$

[3]



$D = 3\,m$

$k = 0.06\ W/mK$

$\delta = 250\ mm$

(a) TEMPERATURE PROFILE:

$\nabla^2 T = 0$ IN SPHERICAL COORDS:

$$\frac{1}{r^2}\frac{d}{dr}\left(k\, r^2 \frac{dT}{dr}\right) = 0$$

$$r^2 \frac{dT}{dr} = c_1$$

$$\frac{dT}{dr} = \frac{c_1}{r^2}$$

$$T(r) = -\frac{c_1}{r} + c_2$$

BCs:

$$T(R) = -\frac{c_1}{R} + c_2 = T_g \Rightarrow c_2 = T_g + \frac{c_1}{R} \Rightarrow T(r) = T_g + c_1\left(\frac{1}{R}\right.$$

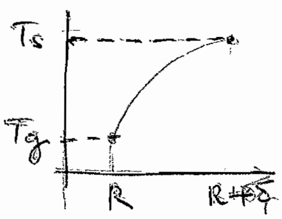
$$-k\frac{dT}{dr}\bigg|_{R+\delta} = -k\frac{c_1}{(R+\delta)^2} = h\left(T(R+\delta) - T_\infty\right)$$

$$= h\left(T_g + \frac{c_1}{R}\left(1 - \frac{R}{R+\delta}\right) - T_\infty\right)$$

$$c_1\left[\left(1 - \frac{R}{R+\delta}\right)\frac{h}{R} + \frac{k}{(R+\delta)^2}\right] = h\left(T_\infty - T_g\right)$$

$$c_1 = \frac{(T_\infty - T_g)}{\frac{1}{R} - \frac{1}{R+\delta} + \frac{k/h}{(R+\delta)^2}}$$

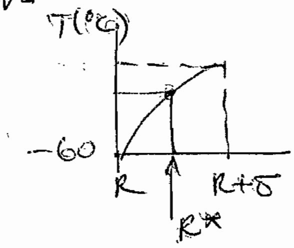
$$T(r) = T_g + \frac{(T_\infty - T_g)}{\frac{1}{R} - \frac{1}{R+\delta} + \frac{k/h}{(R+\delta)^2}}\left(\frac{1}{R} - \frac{1}{r}\right)$$



SUBSTITUTING:

$$T(R^*) = T_g + \frac{(20-(-60))\,K}{\underbrace{\frac{1}{1.5m} - \frac{1}{(1.5+0.25)m}}_{9.85\times10^{-2}\,m^{-1}} + \frac{0.06\,W/mK\,/\,6\,W/m^2K}{(1.5+0.25)^2\,m^2}}\left(\frac{1}{1.5m}-\frac{1}{r}\right)$$

$$0 = T(R^*) = (-60) + 521.4\left(1 - \frac{1.5m}{r}\right)$$

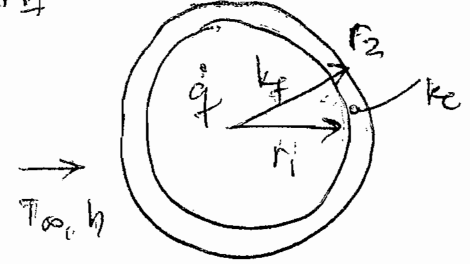


$$\boxed{R^* = 1.69\,m}$$

(1b) THEREFORE, IF THE INSULATION IS PERVIOUS TO MOISTURE, THERE WOULD BE ICE FORMATION INSIDE THE INSULATION

ICE WOULD HAVE A HIGHER THERMAL CONDUCTIVITY THAN THE INSULATION, LEADING TO GREATER HEAT TRANSFER TO THE LIQUEFIED GAS. THEREFORE, AN IMPERMEABLE MOISTURE BARRIER SHOULD BE INSTALLED.

4



(a) INSIDE THE FUEL ROD:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k_f r \frac{\partial T}{\partial r}\right) + \dot{q} = 0$$

$$\frac{\partial}{\partial r}\left[r\frac{dT}{dr}\right] = -\frac{\dot{q}}{k_f} r$$

INTEGRATE:

$$r\frac{dT}{dr} = -\frac{\dot{q} r^2}{2k_f} + C_{f_1} \longrightarrow \frac{dT}{dr} = -\frac{\dot{q} r}{2k_f} + \frac{C_{f_1}}{r}$$

$$T(r) = -\frac{\dot{q} r^2}{4k_f} + C_{f_1} \ln r + C_{f2}$$

B.C: $r\left.\frac{dT}{dr}\right|_{r=0} = -\frac{\dot{q} r^2}{4k_f} + C_{f_1} = 0 \longrightarrow C_{f1} = 0$

$$T(r) = -\frac{\dot{q} r^2}{4k_f} + C_{f2}$$

OUTSIDE FUEL ROD: $\dot{q} = 0$

$$r\frac{dT}{dr} = C_{c_1}$$

$$\frac{dT}{dr} = \frac{C_{c_1}}{r}$$

$$T(r) = C_{c_1} \ln r + C_{c2}$$

B.C: $-k_c\left.\frac{dT}{dr}\right|_{r=r_2} = h(T(r_2) - T_\infty))$

$$-k_c\frac{C_{c_1}}{r_2} = h\left[C_{c_1}\ln r_2 + C_{c2} - T_\infty\right]$$

$$h(T_\infty - C_{c2}) = C_{c_1}\left[h \ln r_2 + \frac{k_c}{r_2}\right]$$

- MATCHING CONDS @ BOUNDARY:

$$q_f(r_1) = q_c(r_1) \qquad (1)$$

$$T_f(r_1) = T_c(r_2) \qquad (2)$$

(1)

$$-k_f \left.\frac{dT}{dr}\right|_{r=r_{1f}} = -k_c \left.\frac{dT}{dr}\right|_{r=r_{1c}}$$

$$-k_f\left[-\frac{\dot{q} r_1}{2k_f}\right] = -k_c \frac{C_{c1}}{r_1} \Rightarrow C_{c1} = -\frac{\dot{q} r_1^2}{2k_c}$$

SOLVING FOR C_{c2}

$$h(T_\infty - C_{c2}) = -\frac{\dot{q} r_1^2}{2k_c}\left[h \ln r_2 + \frac{k_c}{r_2}\right]$$

$$C_{c2} = T_\infty + \frac{\dot{q} r_1}{2h}\left[\frac{h}{k_c} r_1 \ln r_2 + \frac{r_1}{r_2}\right]$$

$$T_c(r) = -\frac{\dot{q} r_1^2}{2k_c} \ln r + \frac{\dot{q} r_1}{2h}\left[\frac{h}{k_c} r_1 \ln r_2 + \frac{r_1}{r_2}\right] + T_\infty$$

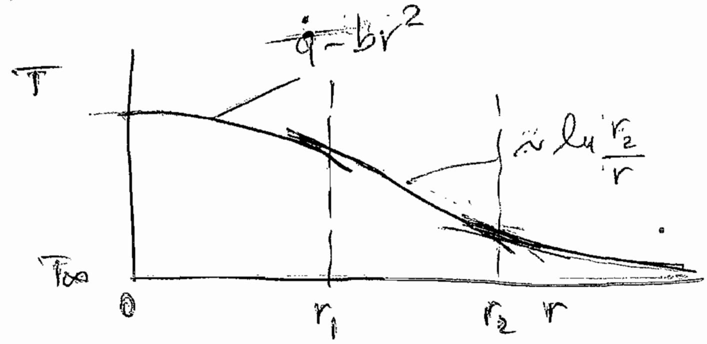
$$= T_\infty + \frac{\dot{q} r_1^2}{2k_c}\left[\ln \frac{r_2}{r}\right] + \frac{\dot{q} r_1}{2h}\frac{r_1}{r_2}$$

SOLVING FOR C_{f2} VIA COND (2):

$$T_f(r_1) = -\frac{\dot{q} r_1^2}{4k_f} + C_{f2} = T_\infty + \frac{\dot{q} r_1^2}{2k_c}\left[\ln\left(\frac{r_2}{r_1}\right)\right] + \frac{\dot{q} r_1}{2h}\frac{r_1}{r_2}$$

$$C_{f2} = T_\infty + \frac{\dot{q} r_1^2}{2k_c}\left[\ln \frac{r_2}{r_1} + \frac{k_c}{h}\frac{1}{r_2}\right] + \frac{\dot{q} r_1^2}{4k_f}$$

$$T_f(r) = T_\infty + \frac{\dot{q} r_1^2}{2k_c}\left[\ln \frac{r_2}{r_1} + \frac{k_c}{h}\frac{1}{r_2}\right] + \frac{\dot{q} r_1^2}{4k_f}\left[1 - \left(\frac{r}{r_1}\right)^2\right]$$



$$T(r_2) = T_\infty + \frac{\dot{q} r_1^2}{2h} \frac{1}{r_2}$$

$$T(r_1) = T_\infty + \frac{\dot{q} r_1^2}{2k_c} \ln \frac{r_2}{r_1} + \frac{\dot{q} r_1^2}{2h r_2} = T(r_2) + \frac{\dot{q} r_1^2}{2k_c} \ln \frac{r_2}{r_1}$$

$$T(0) = T(r_1) + \frac{\dot{q} r_1^2}{4k_f}$$

(b) $k_f = 2\,W/mK$ $\quad$ $k_c = 25\,W/mK$ $\quad$ $\dot{q} = 2\times10^8\,W/m^2$

$r_1 = 6\,mm$ $\quad$ $r_2 = 9\,mm$ $\quad$ $h = 2000\,W/m^2K$

$T_\infty = 300\,K$

SUBSTITUTING

$$\frac{\dot{q} r_1^2}{2h r_2} = \frac{(2\times10^8\,W/m^3)(6\times10^{-3}\,m)^2}{2(2000\,W/m^2K)(9\times10^{-3}\,m)} = 200\,K$$

$$\frac{\dot{q} r_1^2}{2k_c} \ln \frac{r_2}{r_1} = \frac{(2\times10^8\,W/m^3)(6\times10^{-3}\,m)^2}{2(25\,W/mK)} \ln\left(\frac{9}{6}\right) = 58.4\,K$$

$$\frac{\dot{q} r_1^2}{4k_f} = \frac{(2\times10^8\,W/m^3)(6\times10^{-3}\,m)^2}{4(2\,W/mK)} = 900\,K$$

$T(r_2) = 300 + 200 = 500\,K$
$T(r_1) = 500 + 58.4 = 558.4\,K$
$T(0) = 558.4 + 900 = 1458.4\,K$

(c) RECALCULATE THE VALUES OF T(0) FOR VARYING h:

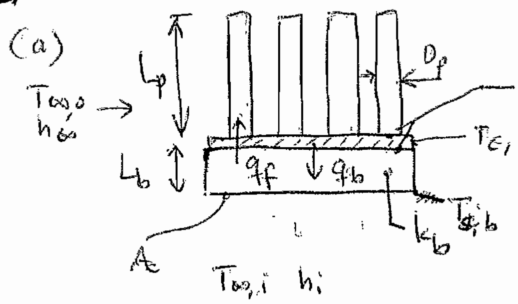
h (W/m²K)	T(0) (K)
2000	1458
5000	1338
10000	1298
100,000	1262
1×10^6	1258

$$\lim_{h\to\infty} T_f(r) = T_\infty + \frac{\dot{q} r_1^2}{2k_c} \ln\frac{r_2}{r_1} + \frac{\dot{q} r_1^2}{4k_f} = 300 + 58.4 + 900\,K = 1258.4$$

THEREFORE, CHANGING THE CONVECTION COEFFICIENT CANNOT BRING THE TEMPERATURES BELOW 1200K.

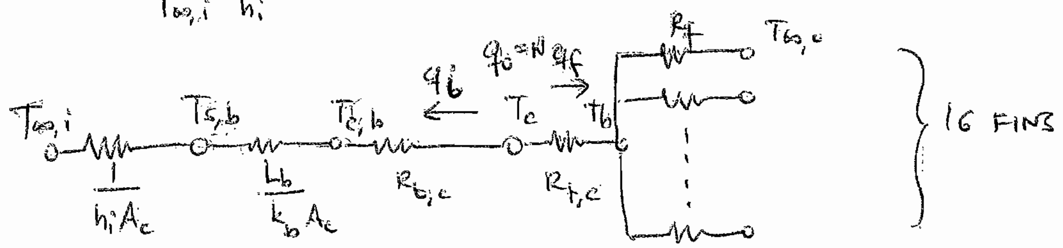
5

(a)



$L_b = 5\,mm$
$k_b = 1\,W/mK$
$R''_{t,c} = 10^{-4}\,m^2K/W$
$k_p = 401\,W/mK$
$D_p = 1.5\,mm$
$L_p = 15\,mm$
$T_{c,MAX} = 75°C$

$T_{\infty,i} = 20°C$
$T_{\infty,o} = 20°C$
$h_i = 40\,W/m^2K$
$h_o = 1000\,W/m^2$
$W = 12.7\,mm$
$A_c = W^2$



$$q_f = M \frac{\sinh mL_p + (h_o/mk_p)\cosh mL_p}{\cosh mL_p + (h_o/mk_p)\sinh mL_p} \qquad R_f = \frac{q_f}{\theta_b}$$

$$m = \sqrt{h_o P / k A_p} \qquad M = \sqrt{h_o h k A_p}\,\theta_b \qquad \theta_b = T_b - T_\infty$$

(b) $q_c = q_o + q_i = q_i + N q_f$

$$q_i = \frac{T_c - T_{\infty,i}}{\frac{L_b}{k_b A_c} + \frac{1}{h_i A_c} + \frac{R''_{t,c}}{A_c}}$$

$$A_c = W^2 = (12.7\times10^{-3}\,m)^2 = 1.613\times10^{-4}\,m^2$$

$$R_b A_c = \underbrace{\frac{5\times10^{-3}\,m}{1\,W/mK}}_{5\times10^{-3}} + \underbrace{\frac{1}{40\,\frac{W}{m^2K}}}_{0.025} + 10^{-4}\,\frac{m^2K}{W} = 3.01\times10^{-2}\,\frac{m^2K}{W}$$

$$R_b = 186.6\,\frac{K}{W}$$

$$q_i = \frac{(75-20)\,K}{186.6\,\frac{K}{W}} = 0.295\,W$$

$$P = \pi D = \pi(1.5\times10^{-3}\,m) = 4.712\times10^{-3}\,m$$

$$A_p = \frac{\pi D^2}{4} = \frac{\pi(1.5\times10^{-3}\,m)^2}{4} = 1.767\times10^{-6}\,m^2$$

$$m = \sqrt{\frac{h_o P}{k_p A_p}} = \left[\frac{(1000\,W/m^2K)(4.71\times10^{-3}\,m)}{(401\,W/mK)(1.767\times10^{-6}\,m^2)}\right]^{1/2} = 81.55\,m^{-1}$$

$$mL_p = 1.223 \qquad \frac{h_o}{mk_p} = \frac{(1000\,W/m^2K)}{(81.55\,m^{-1})(401\,W/m\,K)} = 3.058\times10^{-2}$$

$$\frac{M}{\theta_b} = \sqrt{h_o P k_p A_p} = \sqrt{(1000\, W/m^2K)(4.71\times10^{-3} m)(401\, W/mK)(1.767\times10^{-6} m^2)} =$$
$$= 5.77\times10^{-2}\ \frac{W}{K}$$

$$q_f = \left(5.77\times10^{-2}\frac{W}{K}\right)\underbrace{\frac{\sinh(1.223) + (3.058\times10^{-2})\cosh(1.223)}{\cosh(1.223) + (3.058\times10^{-2})\sinh(1.223)}}_{0.8544}(75-20)\,K$$

$\sinh(1.223) = 1.552$

$\cosh(1.223) = 1.846$

$\underline{q_f = 2.712\, W}$

$$q_c = q_i + N q_f = [0.295 + 16(2.712)]\,W = \underline{43.68\, W} \quad \text{MAX}$$

(d) COMPARE WITH CASE W/O FIN:

$q_i = 0.295\, W$

$$q_{0,o} = h_o A_c (T - T_{\infty,o}) = (1000\, W/m^2K)(1.613\times10^{-4} m^2)(75-20)\,K$$
$$= 8.872\, W$$

$$q_{c,o} = (0.295 + 8.872)\,W = 9.167\, W$$

$$\epsilon = \frac{q_c}{q_{c,o}} = \frac{43.68}{9.167} = 4.77$$

∴ THE HEAT TRANSFER IS ENHANCED BY ALMOST FIVE TIMES.

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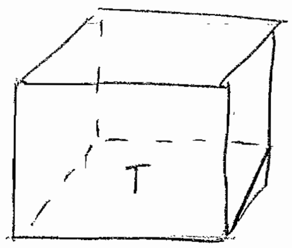


$T_0 = T_\infty$

(a) assuming uniform temperature

$$\rho c V \frac{dT}{dt} = Q - h(6A)(T - T_\infty)$$

$$\frac{dT}{dt} = -\frac{6hA}{\rho c V}(T - T_\infty) + \frac{Q}{\rho c V}$$

$$T(0) = T_0 = T_\infty$$

$$\theta = T - T_0$$

$$\frac{d\theta}{dt} + \frac{\theta}{\tau_a} = \frac{Q}{\rho c V} \qquad \tau_a = \frac{\rho c V}{6hA}$$

(i) $\theta = \theta_\infty \left(1 - e^{-t/\tau_a}\right)$

$$\theta_\infty = \frac{Q}{\rho c V}\tau_a = \frac{Q}{6hA} = \frac{q}{6h}$$

(see attached plot)

(ii) $\theta_\infty = 333\ \mathrm{K}$

(iii) $\tau_a = 2000\ \mathrm{s}$.

6 Cont'd

Xmas cake problem

$A := 0.1 \cdot 0.1 m^2$

$\rho := 200 \frac{kg}{m^3}$

$c := 3000 \frac{J}{kg \cdot K}$

$L := 5cm$

$V := A \cdot (2 \cdot L) \quad V = 1 \times 10^{-3} m^3$

$q := 10^4 \frac{W}{m^2}$ $\quad h := 5 \frac{W}{m^2 K}$

$Q := q \cdot A \quad Q = 100\ W$

ii. and iii.

$\beta := 6 \cdot h \cdot \frac{A}{(\rho \cdot c \cdot V)} \quad \tau_a := \frac{1}{\beta} \quad \tau_a = 2 \times 10^3 s$

$\theta_\infty := \frac{q}{6 \cdot h} \quad \theta_\infty = 333.333\ K$

$\theta_a(t) := \theta_\infty \cdot (1 - \exp(-\beta \cdot t))$

$k := 0.6 \frac{W}{m \cdot K}$

$\alpha := \frac{k}{(\rho \cdot c)} \quad \alpha = 1 \times 10^{-6} \frac{m^2}{s}$

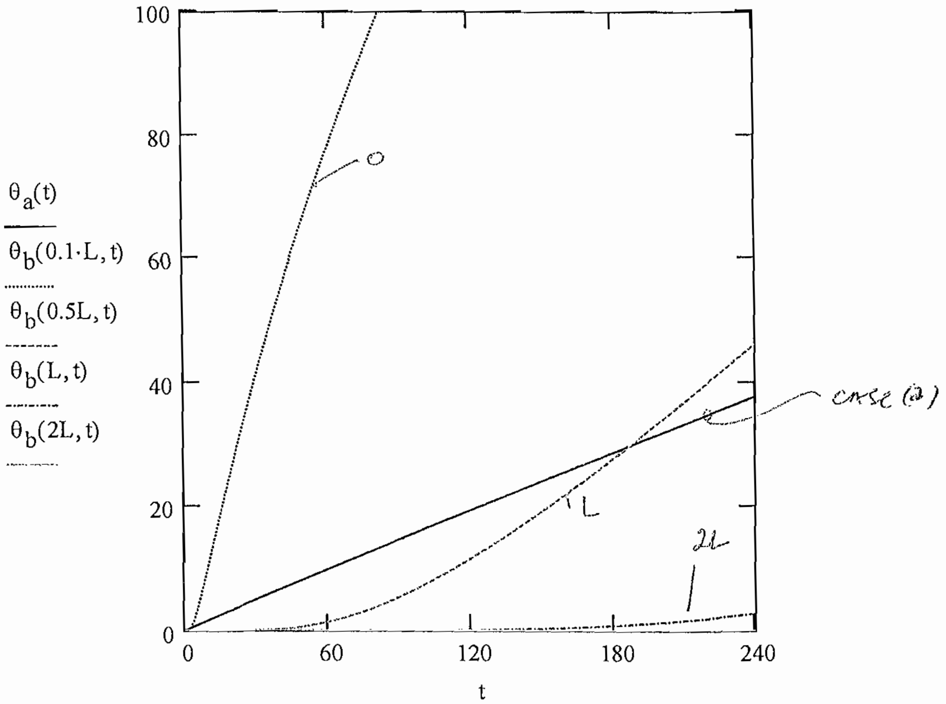
$Bi := h \cdot \frac{L}{k} \quad Bi = 0.417$

iv. Since Bi is not << 1, lumped parameter model may not so good for cube.

(b) i.

$$\theta_b(x,t) := 2\cdot\frac{q}{k}\cdot\left[\alpha\cdot\frac{t\cdot(1.s)}{\pi}\right]^{\frac{1}{2}}\cdot\exp\left[\frac{-\left[(x\cdot 1.\cdot m)^2\right]}{4\cdot\alpha\cdot t\cdot 1.\cdot s}\right] - \frac{q\cdot(x\cdot 1.\cdot m)}{k}\cdot\left[1-\mathrm{erf}\left[\frac{x\cdot 1\cdot m}{\sqrt{4\cdot\alpha\cdot(t)\cdot 1.\cdot s}}\right]\right]$$

$$\tau_b := 2\frac{L}{\alpha} \qquad \tau_b = 1\times 10^5\,\frac{s}{m}$$



ii. The analysis shows that heating using the toaster oven temperatures will greatly exceed 100 K that about 160 seconds, when the temperature rise in the middle of the cake is 20 K. So it would probably burn.

iii. Neglecting convection from the edge of the cake is reasonable, since the temperature at 2L is still very low.

However, the bottom of the cake will be very hot, and therefore, natural convection will take place, smoothing out the temperature profiles.

As long as the characteristic time is much longer than the time considered $t \ll \tau_{b}$ or Fourier number Fo = $t/\tau_b \ll 1$, the semi-infinite solid analysis is valid for toaster oven

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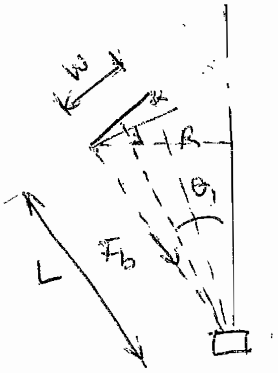
$\theta_1 = 30^\circ \quad \theta_2 = 60^\circ \quad L = 3\,m \quad W = 30\,mm$

$A_p = 0.007\,m^2 \quad T_h = 3000\,K$

INTENSITY OF RADIATION EMITTED BY HEATER (BLACK, DIFFUSE):



$$I_b = \frac{E_b}{\pi} = \frac{\sigma T_h^4}{\pi}$$

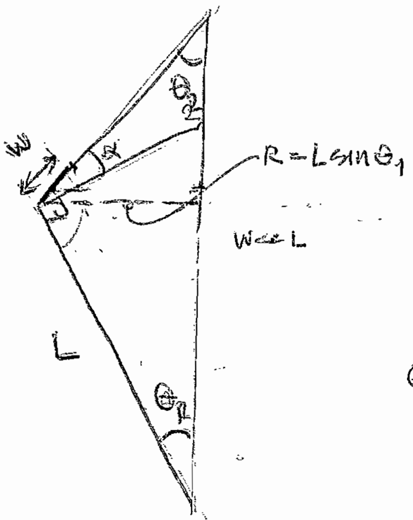
RADIATION INCIDENT ON AREA, ASSUMED TO BE SMALL AND REPRESENTED BY INCIDENCE @ CENTER:

$$G = \int_{\Delta\omega} \underbrace{I_b \cos\theta}_{\text{NORMAL INCIDENCE}} \, d\omega = \int_{\Delta\omega} I_b \cos\theta \frac{dA}{r^2} \approx I_b \cos\theta_1 \,\Delta\omega$$

THE SOLID ANGLE $\Delta\omega$ CAN BE CALCULATED AS: $\dfrac{\text{PROJECTED AREA OF RIN}}{L^2}$

$$\Delta\omega = \frac{2\pi R W \cos\alpha}{L^2}$$

$$\alpha = \frac{\pi}{2} - (\theta_1 + \theta_2) \rightarrow \cos\alpha = \sin(\theta_1 + \theta_2)$$



$$\Delta\omega = \frac{2\pi R W}{L^2} \sin(\theta_1 + \theta_2)$$

$$= \frac{2\pi L W}{L^2} \sin\theta_1 \sin(\theta_1 + \theta_2)$$

$$G = \frac{\sigma T_h^4}{\pi} \cdot \frac{2\pi W}{L} \sin\theta_1 \sin(\theta_1 + \theta_2)$$

$$q = A_p G = A_p \, 2 \frac{W}{L} \sin\theta_1 \sin(\theta_1 + \theta_2) \, \sigma T_h^4$$

$$q = (0.007\,m^2)\, 2 \left(\frac{0.03\,m}{3\,m}\right) \sin 30^\circ \underbrace{\sin(30^\circ + 60^\circ)}_{1} \left(5.669 \times 10^{-8} \frac{W}{m^2 K^4}\right) (3000\,K)^4$$

$$\underline{q = 321\,W}$$

8

(a) STARTING FROM

$$E_{\lambda,b}(\lambda,T) = \frac{c_1}{\lambda^5\left(\exp\left(\frac{c_2}{\lambda T}\right)-1\right)}$$

For $c_2/\lambda T \gg 1 \rightarrow \exp(c_2/\lambda T) \gg 1$

$$E_{\lambda,b}(\lambda,T) = \frac{c_1}{\lambda^5 \exp(c_2/\lambda T)}$$

(b) FOR $c_2 \ll \lambda T$

$$E_{\lambda,b}(\lambda,T) = \frac{c_1 \exp(-c_2/\lambda T)}{\lambda^5\left(1-\exp(-c_2/\lambda T)\right)} =$$

$$\approx \frac{c_1}{\lambda^5}\frac{1}{1-\left(1-\frac{c_2}{\lambda T}\right)} = \frac{c_1}{c_2}\frac{T}{\lambda^4}$$

(c) THE FRACTIONAL ERROR IS:

$$\epsilon = \frac{\frac{c_1}{\lambda^5}\exp(-c_2/\lambda T) - \frac{c_2}{\lambda^5(\exp(c_2/\lambda T)-1)}}{\frac{c_1}{\lambda^5(\exp(c_2/\lambda T)-1)}} = e^{-\eta}(e^{\eta}-1)-1$$

$$\epsilon = 1 - e^{-\eta} - 1 = -e^{-\eta}$$

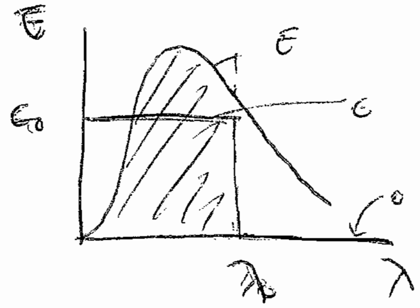
WHERE $\eta = \frac{c_2}{\lambda T}$

For $\lambda T = c_3 \rightarrow \eta = \frac{c_2}{c_3} = 4.965$

$$\epsilon = -e^{-4.965} = 6.98 \times 10^{-3}$$

THEREFORE, very small error @ $\lambda = \lambda(E_{MAX})$

(d) $E_{b,0} = \int_0^\infty \epsilon_\lambda(\lambda) \frac{c_1}{\lambda^5} e^{-c_2/\lambda T} d\lambda$



$$E_{b,0} = \int_0^{\lambda_0} \epsilon_0 \frac{c_1}{\lambda^5} e^{-c_2/\lambda T} d\lambda$$

$$\eta = c_2/\lambda T \qquad \lambda = c_2/\eta T \qquad \frac{d\lambda}{d\eta} = -\frac{c_2}{T}\frac{1}{\eta^2} \qquad \eta_0 = \frac{c_2}{\lambda_0 T}$$

$$E_{b_0} = \epsilon_0 c_1 \int_\infty^{\eta_0} \left(\frac{\eta T}{c_2}\right)^5 e^{-\eta}\left(-\frac{c_2}{T}\frac{1}{\eta^2}\right) d\eta$$

$$= \epsilon_0 c_1 \int_{\eta_0}^\infty \frac{T^4}{c_2^4} \eta^3 e^{-\eta} d\eta$$

$$\int x^3 e^{-x} dx = x^3 \frac{e^{-x}}{(-1)} - \int 3x^2 \frac{e^{-x}}{(-1)} dx = -x^3 e^{-x} + 3x^2 \frac{e^{-x}}{(-1)} - \int 6x \frac{e^{-x}}{(-1)} dx$$

$$= -x^3 e^{-x} - 3x^2 e^{-x} + 6x \frac{e^{-x}}{(-1)} - \int 6 \frac{e^{-x}}{(-1)} dx$$

$$= -x^3 e^{-x} - 3x^2 e^{-x} - 6x e^{-x} + 6 \frac{e^{-x}}{(-1)} =$$

$$= -(x^3 + 3x^2 + 6x + 6) e^{-x}$$

$$E_{b,0} = \frac{\epsilon_0 c_1 T^4}{c_2^4} \left[-(\eta^3 + 3\eta^2 + 6\eta + 6) e^{-\eta} \right]_{\eta_0}^\infty =$$

$$= \frac{\epsilon_0 c_1 T^4}{c_2^4} \left[0 + (\eta_0^3 + 3\eta_0^2 + 6\eta_0 + 6) e^{-\eta_0} \right]$$

$E_{b,0} = 3.781 \times 10^3 \ W/m^2$

$F_{0\to\lambda_0} = E_{b,0}/E_b = E_{b0}/\sigma T^4 = 0.067 \rightarrow$ very good agreement with table for $F_{0\to\lambda}$