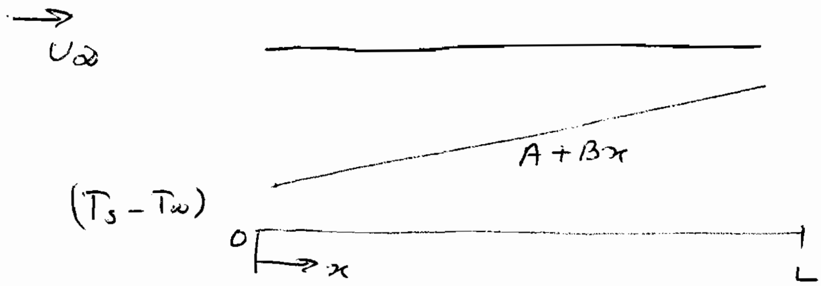
Example paper - 3 3A6 - CRIBS

(i)



using the principle of super-position

$$\frac{\dot{q}_s}{\dot{q}_s^*(T_s)} = \int_0^x \left[1 - (\xi/x)^{3/4}\right]^{-1/3} \frac{dT_s}{d\xi}\, d\xi \, + A$$

$$T_s = T_\infty + A + B\xi \quad \Rightarrow \quad \frac{dT_s}{d\xi} = B$$

Let $s = 1 - (\xi/x)^{3/4} \Rightarrow ds = -\frac{3}{4x}(\xi/x)^{+1/4} d\xi$

$$\therefore \frac{\dot{q}_s}{\dot{q}_s^*(T_s)} = B \int_0^x s^{-1/3}(1-s)^{1/3}\left(\frac{4}{3}x\right) ds \quad + A$$

$$= \frac{4}{3} Bx \int_0^1 s^{-1/3}(1-s)^{1/3}\, ds$$

use β- function integrals

$$\int_0^1 z^{m-1}(1-z)^{n-1}\, dz = \beta(m,n)$$

for us, $m = 2/3$, $n = 4/3 \Rightarrow \beta(2/3, 4/3) = 1.209$

$$\therefore \dot{q}_s / \dot{q}_s^*(T_s) = \left(\frac{4}{3} * 1.209\, Bx + A\right)$$

$$\frac{\dot{q}_s}{\dot{q}_s^*(T_s)} = 1.612 B x + A.$$

$\dot{q}_s^*(T_s)$ - heat transferred when the surface temp. is uniform @ T_s.

which is given by

$$\dot{q}_s^*(T_s) = h(T_s - T_\infty) = \frac{k_f}{x} Nu_x (T_s - T_\infty)$$

for laminar flow $Nu_x = 0.332\, P_r^{1/3}\, Re_x^{1/2}$.

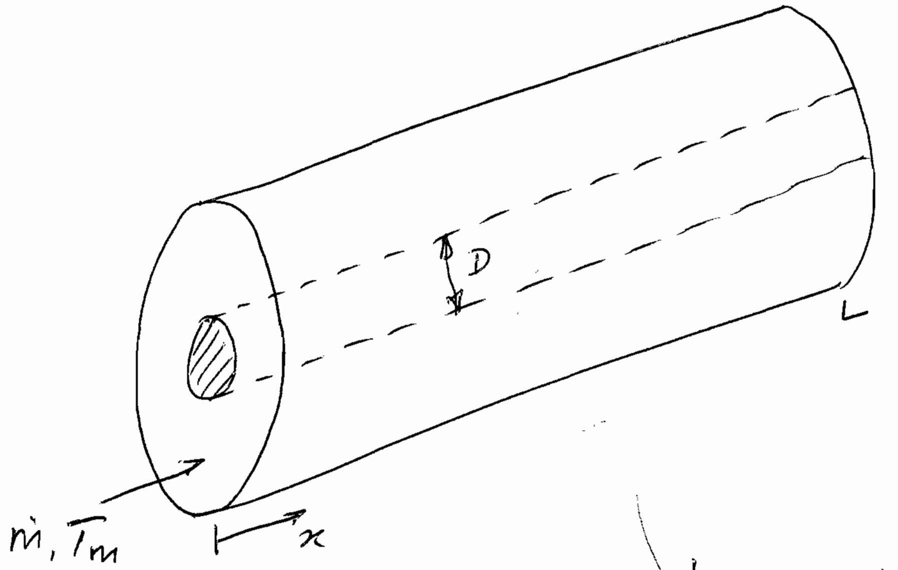
$$\therefore \boxed{\dot{q}_s = \frac{0.332\, k_f}{x} Re_x^{1/2}\, P_r^{1/3} (T_s - T_\infty) * (1.612 B x + A)}$$

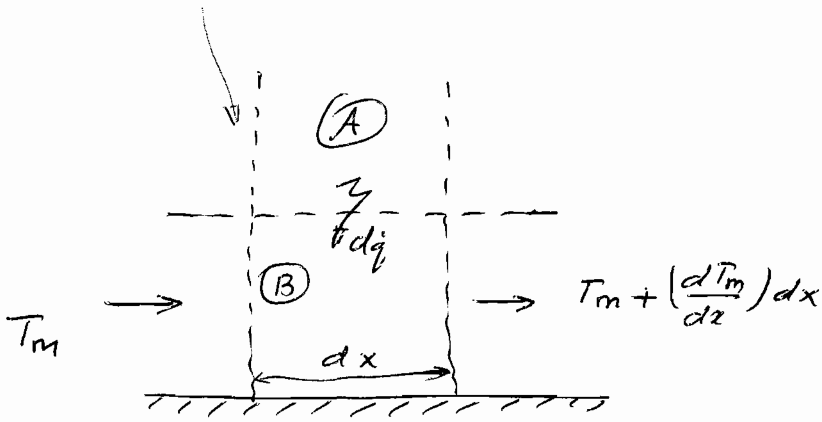
$$\dot{Q}_s = \bar{\dot{q}}_s * L\cancel{w}$$ unity

$$\bar{\dot{q}}_s = \frac{\bar{h} L}{k_f}$$

$$\dot{Q}_s = 0.664 \frac{k_f}{L} Re_L^{1/2}\, P_r^{1/3} (T_s - T_\infty) * (1.612 BL + A)$$

(2)





a) using control volume (A)

energy generation = energy out.

$$\dot{Q}\,\frac{\pi D^2}{4}\,dx = d\dot{q}_s = \dot{q}_s\,\pi D\,dx$$

$$\therefore \quad \dot{q}_s = \frac{\dot{Q} D}{4} = \left(\frac{Q_o D}{4}\right)\sin\left(\frac{\pi x}{L}\right)$$

$$\boxed{\dot{q}_s = \left(\frac{Q_o D}{4}\right)\sin\left(\frac{\pi x}{L}\right)}$$

Total heat transferred is

$$\dot{q} = \int_0^L \dot{q}_s\,dA = \int_0^L \dot{q}_s\,\pi D\,dx$$

$$= \pi\,\frac{Q_o D^2}{4}\int_0^L \sin\left(\frac{\pi x}{L}\right)dx$$

$$\boxed{\dot{q} = \frac{Q_o D^2 L}{2}}$$

b) Consider the control volume B

$$\cancel{d}\dot{q}_s \pi D\, dx - \dot{m} C_p \left(\frac{dT_m}{dx}\right) dx = 0 .$$

$$\Rightarrow \quad \frac{dT_m}{dx} = \frac{\dot{q}_s \pi D}{\dot{m} C_p} = \left(\frac{\pi Q_o D^2}{4 \dot{m} C_p}\right) \sin\left(\frac{\pi x}{L}\right)$$

$$\Rightarrow \quad T_m(x) - T_m(x=0) = \left(\frac{\pi Q_o D^2}{4 \dot{m} C_p}\right) \int_0^x \sin\left(\frac{\pi x}{L}\right) dx$$

$$\Rightarrow \quad T_m(x) = T_m(x=0) + \left(\frac{\pi Q_o D^2 L}{4 \dot{m} C_p \pi}\right)\left(1 - \cos\left(\frac{\pi x}{L}\right)\right)$$

$$\boxed{T_m(x) = T_m(0) + \frac{Q_o L D^2}{4 \dot{m} C_p}\left[1 - \cos\left(\frac{\pi x}{L}\right)\right]}$$

c) At the interface of control volumes A & B

$$\dot{q}_s = h_1 (T_s - T_m) \Rightarrow \boxed{T_s = \frac{\dot{q}_s}{h_1} + T_m}$$

T_s is maximum when $\frac{dT_s}{dx} = 0$

which happens @ $x = \frac{L}{\pi} \tan^{-1}\left(-\frac{\dot{m} C_p}{h_1 L D}\right)$

$$T_s(x) = \left(\frac{Q_o D}{4h_1}\right) \sin\left(\frac{\pi x}{L}\right) + T_m(0) + \left(\frac{Q_o L D^2}{4\dot{m}C_p}\right)\left[1 - \cos(x)\right]$$

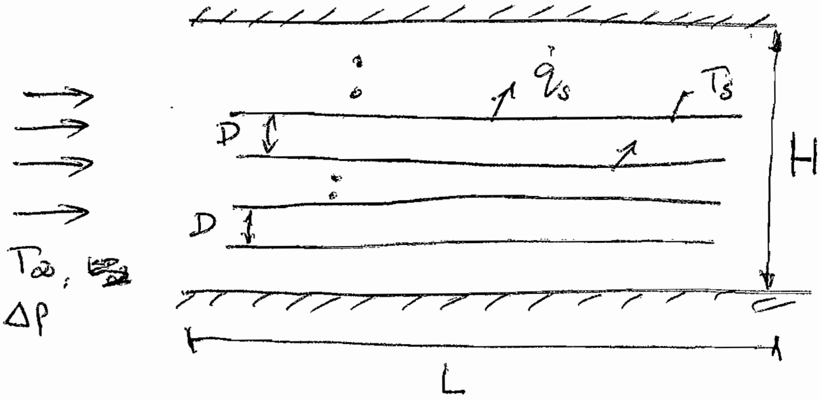
$$\frac{dT_s}{dx} = \frac{Q_o \cancel{D} \cancel{\pi}}{\cancel{4} h_1 \cancel{L}} \cos\left(\frac{\pi x}{L}\right) + \frac{Q_o L D^{\cancel{2}} \cancel{\pi}}{\cancel{4}\dot{m}C_p \cancel{L}} \sin\left(\frac{\pi x}{L}\right) = 0$$

$$= \frac{1}{h_1} \cos\left(\frac{\pi x}{L}\right) + \frac{LD}{\dot{m}C_p} \sin\left(\frac{\pi x}{L}\right) = 0$$

$$\Rightarrow \quad \frac{\sin\left(\frac{\pi x}{L}\right)}{\cos\left(\pi x/L\right)} = -\frac{\dot{m}C_p}{h_1 L D} = \tan\left(\frac{\pi x}{L}\right)$$

$$\therefore \quad \boxed{x = +\frac{L}{\pi} \tan^{-1}\left(-\frac{\dot{m}C_p}{h_1 L D}\right)}$$

3 Optimal Spacing: (Forced Convection)



What is D_{opt} so that $\dot{q}_s$ is max.?

Sol: n – # of boards $\sim \left(\frac{H}{D}\right) >> 1$

Two limit are possible.

i) Fully developed flow

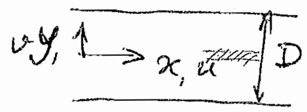
ii) B' layer flow

To have fully developed flow

$$\delta_T \sim (D/2) \quad \text{or} \quad D \to 0.$$

$$\dot{Q} = \dot{m}\, c_p (T_s - T_\infty); \qquad \dot{m} = \rho U A = \rho H W U$$

Channel flow:



$$\frac{dp}{dx} = \mu \frac{\partial^2 u}{\partial y^2} \qquad u = 0 \ @ \ y = \pm D/2$$

$$\frac{du}{dy} = \frac{1}{\mu}\left(\frac{dp}{dx}\right) y + \cancel{c_1}^{\,0}$$

$$u(y) = \frac{1}{2\mu}\left(\frac{dp}{dx}\right) y^2 + c_2$$

$$c_2 = -\frac{1}{8\mu}\left(\frac{dp}{dx}\right) D^2$$

$$\therefore u(y) = \frac{D^2}{8\mu}\left(\frac{dp}{dx}\right)\left[1 - \left(\frac{y}{D/2}\right)^2\right]$$

$$\dot{m} = \int_{-D/2}^{D/2} \rho\, u\, dA = \rho W \int_{-D/2}^{D/2} \frac{D^2}{8\mu}\left(-\frac{dp}{dx}\right)\left[1 - \left(\frac{y}{D/2}\right)^2\right] dy$$

$$y - \frac{4}{D^2}\frac{y^3}{3}\Big|_{-D/2}^{D/2} \qquad \underbrace{2\times 2\times 2}_{8}$$

$$D/2 - \frac{4}{D^2}\frac{D^3}{24} + D/2 - \frac{4}{D^2}\frac{D^3}{24}.$$

$$D - \frac{\cancel{4}^{1}}{\cancel{12}_{3}}D. = D - D/3 = \frac{2}{3}D$$

$$\therefore \dot{m} = \cancel{\rho W}\frac{D^2}{8\mu}\left(-\frac{dp}{dx}\right)\frac{2}{3}\cancel{D} = \cancel{\rho}\,\bar{v}\,\cancel{DW}.$$

$$\therefore \bar{v} = \frac{2}{3}\frac{D^2}{8\mu}\left(-\frac{dp}{dx}\right) = \frac{D^2}{12\mu}\left(-\frac{dp}{dx}\right)$$

$$\therefore \boxed{u(y) = \frac{3}{2}\bar{v}\left[1 - \left(\frac{y}{D/2}\right)^2\right]}$$

$\rho D W \bar{v}$

$$\therefore \dot{Q}_1 = \cancel{\rho H W}\ \dot{m}\, c_p (T_s - T_\infty) = \left(\frac{\dot{m}}{\text{Channel}}\right) \text{ \# of channel } c_p (T_s - T_\infty)$$

$$= \frac{D^2}{12\mu}\left(-\frac{dp}{dx}\right)(\rho \cancel{D} W) * \frac{H}{\cancel{D}}\, c_p\, (T_s - T_\infty)$$

$$\dot{Q}_1 = \rho c_p (HW)\frac{D^2}{12\mu}\left(\frac{\Delta P}{L}\right)\cancel{c_p}(T_s - T_\infty)$$

$$\Rightarrow \boxed{\dot{Q}_1 \sim D^2}$$

2) B'layer limit: (find U_∞) two sides/board.

Force balance $H\Delta p \approx (\bar{\tau}_w * L) * \cancel{2} * \cancel{n}$ # of boards.

But $\frac{\bar{C}_f}{2} = \frac{\bar{\tau}_w}{\rho U_\infty^2} \Rightarrow \bar{\tau}_w = \frac{\bar{C}_f}{2}\rho U_\infty^2$

But $\frac{\bar{C}_f}{2} = 0.664\, Re_L^{-1/2}$ (from Eq. 3.8)

$$\therefore H\Delta P = 1.328\, Re_L^{-1/2}\rho U_\infty^2 \left(\frac{H}{D}\right) L$$

⑦ ③

$$Re_L^{-1/2} U_\infty^2 = \frac{H\Delta P}{1.328\,\rho\, n L} \qquad Re_L = \frac{UL}{\nu} \qquad \frac{\nu^{1/2}}{U_\infty L^{1/2}}$$

$$\therefore\ U_\infty = \left(\frac{H\Delta P}{1.328\rho n\, L^{1/2}\nu^{1/2}}\right)^{2/3}$$

Now $\dot{Q}_2 = 2nA\bar{q}_s \qquad \bar{q}_s = \bar{h}(T_s - T_\infty)$

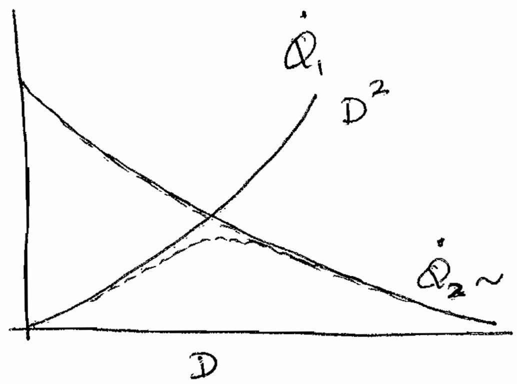
$$\overline{Nu_L} = 2\,Nu_L = 0.664\, Re_L^{1/2} Pr^{1/3} = \frac{\bar{h}L}{k_f}$$

$$\therefore\ \dot{Q}_2 = 2n\,(\cancel{L}W)\frac{k_f}{\cancel{L}}\,0.664\,Pr^{1/3}\left(\frac{U_\infty L}{\nu}\right)^{1/2}(T_s - T_\infty)$$

$$= 1.328\,k_f\,(T_s - T_\infty)\,Pr^{1/3}\,W\,n\,\frac{U_\infty^{1/2}L^{1/2}}{\nu^{1/2}}$$

$$\therefore\ \dot{Q}_2 = 1.328\,k_f\,Pr^{1/3}(T_s - T_\infty)\,W\,n^{2/3}\,\frac{L^{1/2}}{\nu^{1/2}}\left(\frac{H\Delta P}{1.328\,L^{1/2}\nu^{1/2}}\right)^{1/3}$$

$$\Rightarrow\ \dot{Q}_2 \sim n^{2/3} \ \Rightarrow\ \dot{Q}_2 \sim D^{-2/3}$$



$$\frac{D^2}{L}\ \frac{P^{2/3}}{L^{1/3}} = \frac{D_{op}^{2/3}}{L^{4/3}}$$

$$D_{opt} = \left(\frac{L^{4/3}\,\alpha\,\mu}{\Delta P}\right)^{3/8}$$

$$\frac{D_{opt}}{L} = \frac{L^{4/8}}{L}\left(\frac{\alpha\,\mu}{\Delta P}\right) = L^{1/8}$$

$\therefore\ \dot{Q}_{max}$ when $\dot{Q}_1 = \dot{Q}_2$

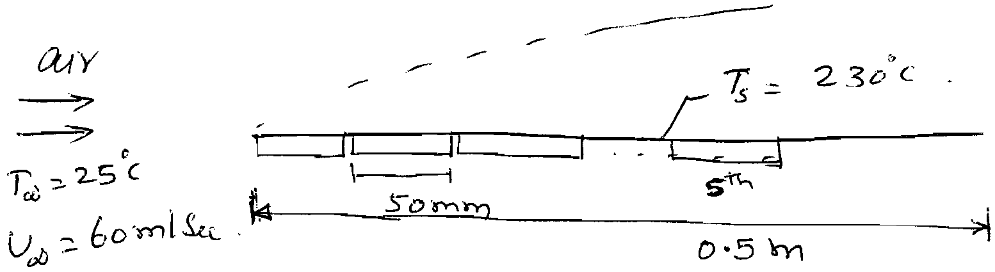
$$\rho\, C_p\,\cancel{(HW)}\,\frac{D_{opt}^2}{12\mu}\left(\frac{\Delta P}{L}\right)\cancel{C_p}\,\cancel{(T_s - T_\infty)} \simeq 1.208\,k_f\,\cancel{(T_s - T_\infty)}\,\cancel{HW}\,\frac{Pr^{1/3}L^{1/3}\Delta P^{1/3}}{\rho^{1/3}\nu^{2/3}\,D_{opt}^{2/3}}$$

$$-\tfrac{+2}{3}\ \tfrac{8}{3}$$

$$\Rightarrow\ \frac{D_{opt}}{L} \simeq (1.208\times 12)^{3/8}\left(\frac{\alpha\,\mu}{\Delta P L^2}\right)^{1/4}$$

$$\dot{q}_{max} \leq 0.62\left(\frac{\rho\Delta P}{Pr}\right)^{1/2}\frac{H\,C_p}{(T_s - T_\infty)} \quad \therefore \quad \boxed{\frac{D_{opt}}{L} \approx 2.73\left(\frac{\mu\alpha}{\Delta P L^2}\right)^{1/4}}$$

(4)



Soln:

film temperature $T_f = \dfrac{T_\infty + T_s}{2} = 127.5°C$.

air properties @ 127.5°C.

$C_p = 1.023 \times 10^3$ J/kg-K

$\mu = 23.1 \times 10^{-6}$ kg/sec-m

$k = 0.034$ W/m-K

$P_r = 0.7$

$\rho = 0.87$ kg/m^3

The flow will become turbulent after some distance x_c. The boundary layer will have both laminar & turbulent part.

To find the location of transition

$$Re_c = 5 \times 10^5 = \frac{U_\infty x_c}{\nu} \Rightarrow x_c = 0.22 \text{ m}$$

ie a portion of 5th heater will have turbulent b'layer.

for laminar flow: $Nu_x = 0.332\, Re_x^{1/2}\, P_r^{1/3}$.

turbulent flow: $Nu_x = 0.038\, Re_x^{4/5}\, P_r^{1/3}$

To get the average heat transfer rate from the fifth strip

$$\bar{q}_5 = \bar{q}_{0-5} - \bar{q}_{0-4}$$

$$\bar{q}_{0-4} = \bar{h} A (T_s - T_\infty)$$

$$\overline{Nu}_{0.4} = \frac{\bar{h} L_{0.4}}{k_f} = 2\, Nu_{0-4} = 0.664\, Re_L^{1/2}\, P_r^{1/3}$$

$$\bar{h} = \frac{0.664\, k_f}{L_{0-4}} Re_L^{1/2} P_r^{1/3}$$

$$L_{0.4} = 4 \times 50 = 200\,m = 0.2\,m$$

$$Re_L = \frac{0.87 * 60 * 0.2}{23.1 \times 10^{-6}} = 4.52 \times 10^5$$

$$\bar{h} = \frac{0.664 * 0.034}{0.2} * (4.52 \times 10^5)^{1/2}\, 0.7^{1/3}$$

$$\bar{h} = 67.38 \quad W/m^2\text{-}K$$

$$\bar{q}_{0-4} = 67.38 * (0.2 * 1)(230 - 25) = 2.76 \times 10^3\, W$$

for 0-5 - the boundary layer is of mixed type

So, $\overline{Nu}_{0-5} = \frac{\bar{h} L_{0-5}}{k_f} = P_r^{1/3} \left\{ 0.664\, Re_{x_c}^{1/2} + 0.038 \left(Re_{L_{0.5}}^{4/5} - Re_{x_c}^{4/5} \right) \right\}$

$L_{0.5} = 0.25\,m$; $Re_{x_c} = 5 \times 10^5$; $\quad Re_{L_{0.5}} = 5.65 \times 10^5$

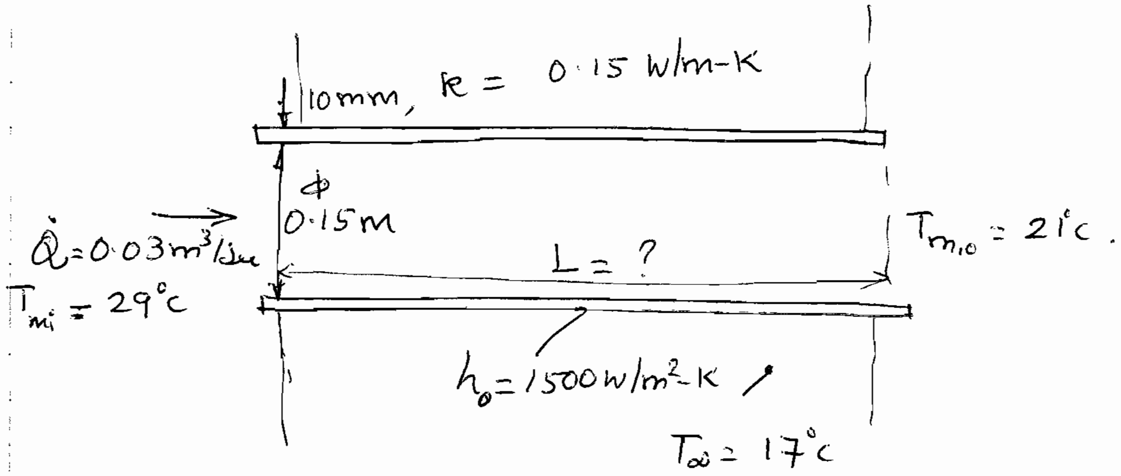
$$\therefore \bar{h} = \frac{k}{L} 0.7^{1/3} \left\{ 0.664 * (5 \times 10^5)^{1/2} + 0.038 \left((5.65 \times 10^5)^{4/5} - (5 \times 10^5)^{4/5} \right) \right\}$$

$$\bar{h} = 73.92 \quad W/m^2\text{-}K$$

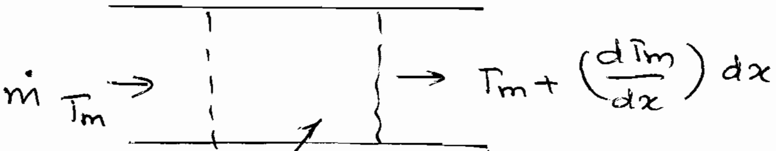
$$\therefore \bar{q}_{0.5} = 73.92 * (0.25 \times 1) * (230 - 25) = 3.79 \times 10^3\, W.$$

$$\therefore \boxed{\bar{q}_5 = 1.03 \times 10^3\, W}$$

5



Sol:



$$\dot{q} = U A_s (T_\infty - T_m) = U \pi D\, dx\, (T_\infty - T_m)$$

$$\Rightarrow \quad \dot{m} C_p \frac{dT_m}{dx} = U \pi D (T_\infty - T_m)$$

$$\therefore \frac{d\Delta T}{\Delta T\, dx} = -\frac{U\pi D}{\dot{m} C_p} \Rightarrow \boxed{\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{UA_s}{\dot{m}C_p}\right)}$$

$$R_{tot} = (UA_s)^{-1} = \frac{1}{h_i \pi D_i L} + \frac{\ln(D_o/D_i)}{2\pi L k} + \frac{1}{h_o \pi D_o L}$$

h_i: $\quad Re_D = \dfrac{4\dot{m}}{\pi D_i \mu} = \dfrac{4 * 1.2 * 0.03}{\pi * 0.15 * 17.9\times10^{-6}} = 17.07\times10^{3}$

The flow is turbulent.

$$\therefore \quad Nu = 0.023\, Re_D^{4/5}\, Pr^{1/3} = \frac{h_i D_i}{k}$$

$$\therefore \quad h_i = \frac{0.026}{0.15} * 0.023 * (17.07\times10^{3})^{4/5} (0.7)^{1/3}$$

$$h_i = 8.61 \ \text{W/m}^2\text{K}$$

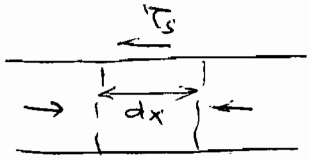
$$\therefore \; R_{total} = \frac{1}{8.61 * \pi * 0.15 * L} + \frac{\ln(0.17/0.15)}{2\pi * 0.15 * L} + \frac{1}{1500 * \pi * 0.17 * L}$$

$$= \frac{0.25}{L} + \frac{0.13}{L} + \frac{1.25\times 10^{-3}}{L} = \frac{0.381}{L}$$

$$\therefore (UA_s) = R_{tot}^{-1} = 2.63\,L \quad W/K.$$

$$\frac{17-21}{17-29} = \exp\left(\frac{-2.63\,L}{0.036 * 1007}\right) \Rightarrow \boxed{L = 15.02\ m}$$

The require fan power is from force balance in fully developed flow:



$$P_o = (\Delta p)\,\dot{Q}$$

$$\frac{dp}{dx} = \tau_s \, P/A_c \qquad \frac{4\pi D_i}{\pi D^2}$$

$$\Rightarrow \Delta p = \tau_s \frac{PL}{A_c}$$

$$= \frac{C_f}{2} \rho U^2 \left(\frac{4}{D}\right) L$$

$$= 2 C_f \left(\frac{\rho U^2}{D_i}\right) L \, \dot{Q}$$

From Moody's diagram

$$4 C_f = 0.029 \quad \text{for} \quad Re = 1.71\times 10^4$$

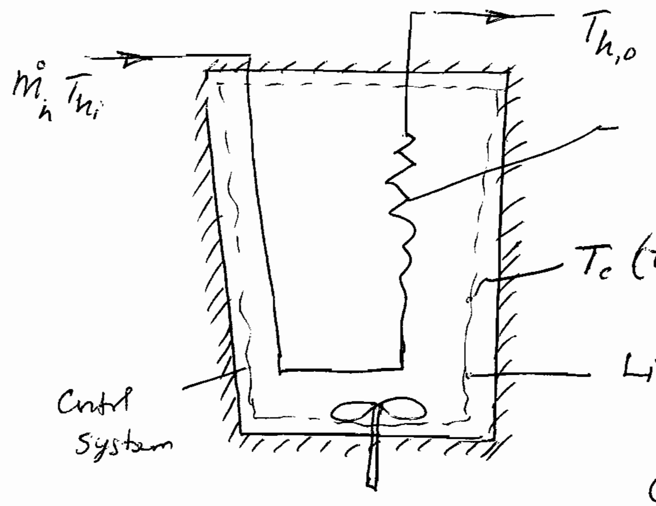
$$U = \frac{4\dot{Q}}{\pi D_i^2} = 1.698 \ m/sec.$$

$$\therefore \; P_o = 0.0145 \left(\frac{1.2 * 1.698^2}{0.15}\right) * 15.02 * 0.03$$

$$\boxed{P_o = 0.151 \ W}$$

entry region $\Big\}$: $\left(\frac{L}{D}\right) = 0.623\, Re_D^{0.25} \Rightarrow$ Assumption of fully developed flow is satisfied..

6)



Sol:

Energy balance $\Delta U = \Delta Q + \cancel{\Delta W}^{\approx 0}$

$$\Rightarrow \frac{dU}{dt} = \dot{q}(t)$$

$$m_c C_{vc} \frac{dT_c}{dt} = \dot{m}_h c_{ph}(T_{h,i} - T_{h,o}) = UA_s \Delta T_{lm}$$

$$\Delta T_{lm} = \frac{\Delta T_{out} - \Delta T_{in}}{\ln\left(\frac{\Delta T_{out}}{\Delta T_{in}}\right)} = \frac{(T_{h,o} - T_c) - (T_{h_i} - T_c)}{\ln\left(\frac{T_{h,o} - T_c}{T_{h_i} - T_c}\right)}$$

$$= \frac{(T_{ho} - T_{h_i})}{\ln\left(\frac{T_{h,o} - T_c}{T_{h_i} - T_c}\right)} = \frac{\dot{m}_h c_{ph}}{UA_s}(T_{h_i} - T_{h,o})$$

$$\Rightarrow \ln\left[\frac{(T_{h,o} - T_c)}{(T_{h_i} - T_c)}\right] = -\frac{UA_s}{\dot{m}_h c_{ph}}$$

$$\therefore \boxed{T_{h,o} = T_c + (T_{h_i} - T_c)\exp\left(-\frac{UA_s}{\dot{m}_h c_{ph}}\right)}$$

$$\Rightarrow q(t) = \dot{m}_h c_{ph}(T_{h_i} - T_c)\left\{1 - \exp\left(-\frac{UA_s}{\dot{m}_h c_{ph}}\right)\right\}$$

$$\therefore m_c C_{vc}\frac{dT_c}{dt} = \dot{m}_h c_{ph}(T_{h_i} - T_c)\left\{1 - \exp\left(-\frac{UA_s}{\dot{m}_h c_{ph}}\right)\right\}$$

$$\frac{dT_c}{(T_{hi}-T_c)\,dt} = \left(\frac{\dot m_h c_{ph}}{\dot m_c C_{vc}}\right)\left\{ \quad \right\}$$

$$\frac{T_{hi} - T_{cf}}{T_{hi} - T_{co}} = Exp\left\{-t\,\frac{\dot m_h c_{ph}\{\quad\}}{\dot m_c\, C_{vc}}\right\}$$

$$\therefore \boxed{T_c = T_{hi} - (T_{hi} - T_{co})\, Exp\left\{- \quad \right\}}$$

To get $\underline{\underline{(UA_s)}}$

$$(UA_s)^{-1} = \frac{1}{h_o A_o} + \frac{1}{h_i A_i} = \frac{1}{A}\left(\frac{1}{h_o} + \frac{1}{h_i}\right)$$

$h_o = 1000$ w/m²-K.

h_i - internal flow @ $Re_D = \frac{4\dot m}{\pi D \mu} = \frac{4*2.4}{\pi*0.05*0.002}$

$= 3.06\times10^4$.

the flow is turbulent.

$$\Rightarrow \overline{Nu} = \frac{\bar h D}{k} = 0.023\, Re_D^{4/5}\, Pr^{1/3}$$

$$\Rightarrow \bar h_i = \frac{0.26}{0.05}\left\{0.023 * (3.06\times10^4)^{4/5} * 20^{1/3}\right\}$$

$= 1258.9$ w/m² K.

$$\therefore U^{-1} = \frac{1}{1258.9} + \frac{1}{1000} \Rightarrow \underline{\underline{U = 557.31 \text{ W/m}^2\text{K}}}$$

Can be integrated to yield

$$\boxed{T_c = T_{hi} - (T_{hi} - T_{c,i})\, Exp\left\{ -t\, \frac{\dot{m}_h c_{p_h} \left[1 - \exp\left(-\frac{UA_s}{\dot{m}_h c_{p_h}}\right)\right]}{\rho_c V c_{v_c}} \right\}}$$

<u>To get U:</u>

$$U^{-1} = \left(\frac{1}{h_o} + \frac{1}{h_i}\right)$$

$h_o = 1000$ W/m²-K.

$Re_D = 3.06 \times 10^4$, The internal flow is turbulent.

$$\Rightarrow h_i = \frac{k}{D}\, 0.023\, Re_D^{4/5}\, P_r^{1/3}$$

$$h_i = 1258.9 \text{ W/m}^2\text{-K}.$$

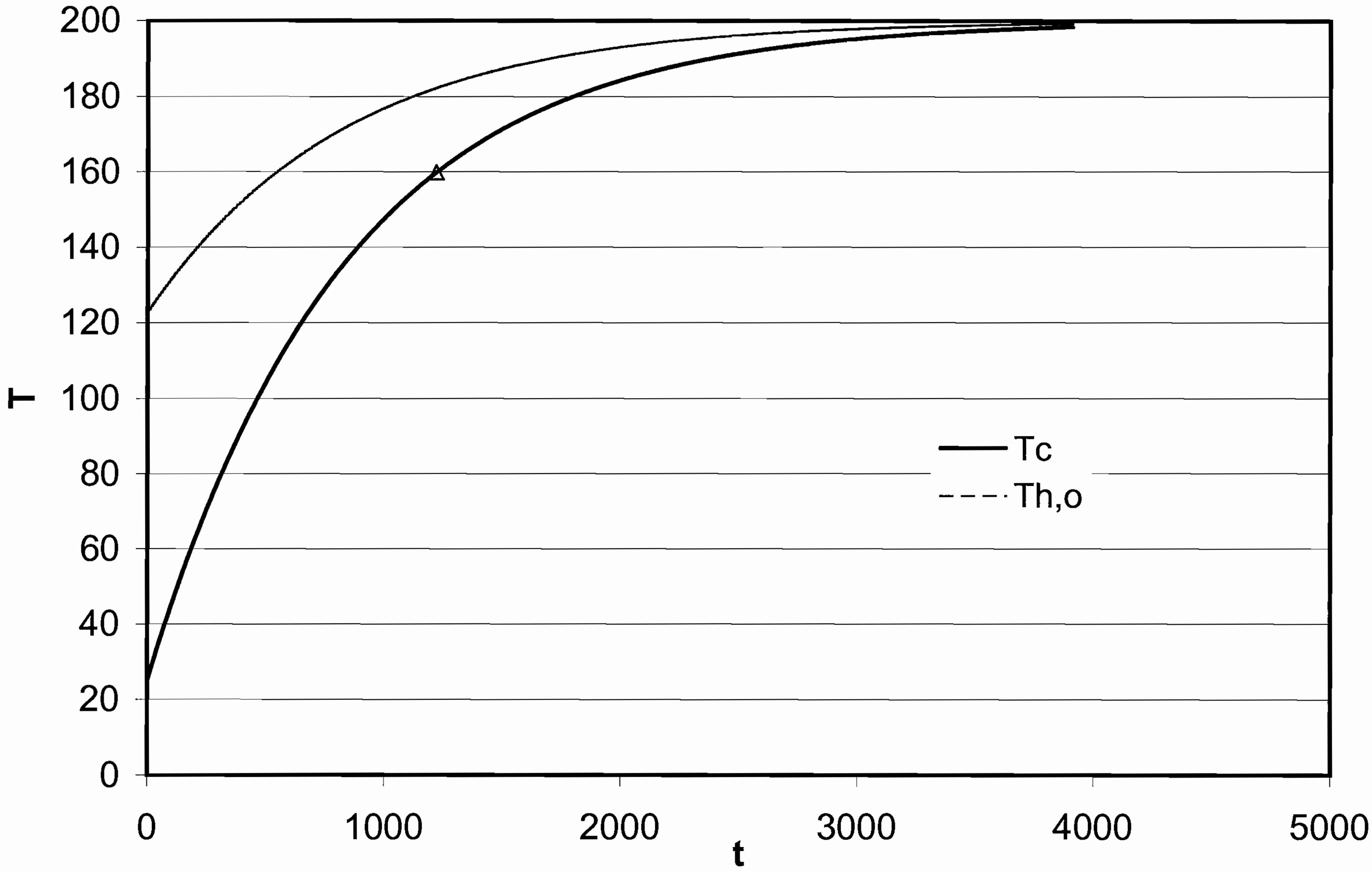
$$\therefore \quad U = 557.31 \text{ W/m}^2\text{-K}.$$

<u>From the graph attached:</u> (obtained using spread sheet)

$t_1 = 1220$ Sec to achieve $T_c = 160°C$

if $\dot{m}_h \uparrow \quad h_i \uparrow \Rightarrow U \uparrow \Rightarrow t_1 \downarrow$.

$T_{hi} \uparrow \quad q(t) \uparrow \Rightarrow t_1 \downarrow$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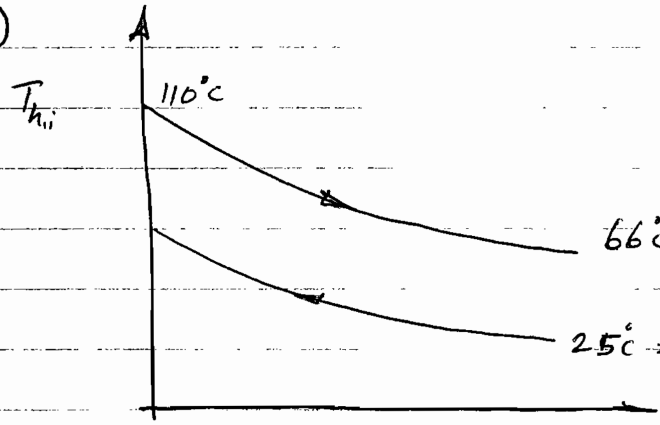


$C_h = 2166$ J/kg K

$C_c = 4178$ J/kg K.

0.1 kg/Sec.

$\dot{m} = 0.2$ kg/Sec.

$A = 5\ m^2$, $U = 38\ W/m^2 K$

Balance on the hot side:

$$\dot{q} = \dot{m}_h C_h (T_{h_i} - T_{h,o}) = 0.1 \times 2166 (110 - 66)$$

$$= 9530\ W$$

On the cold side:

$$9530 = \dot{m}_c C_c (T_{c_o} - T_{c,i}) \Rightarrow T_{c,o} = 36.4\ °c$$

To find if Fouling has occurred:

$$\dot{q} = UA\,\Delta T_{lm} \qquad \Delta T_{lm} = \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o/\Delta T_i)}$$

$$\left.\begin{aligned} \Delta T_o &= 66 - 25 = 41\ °c \\ \Delta T_i &= 110 - 36.4 = 73.6\ °c. \end{aligned}\right\} \Rightarrow \Delta T_{lm} = 55.7\ °c$$

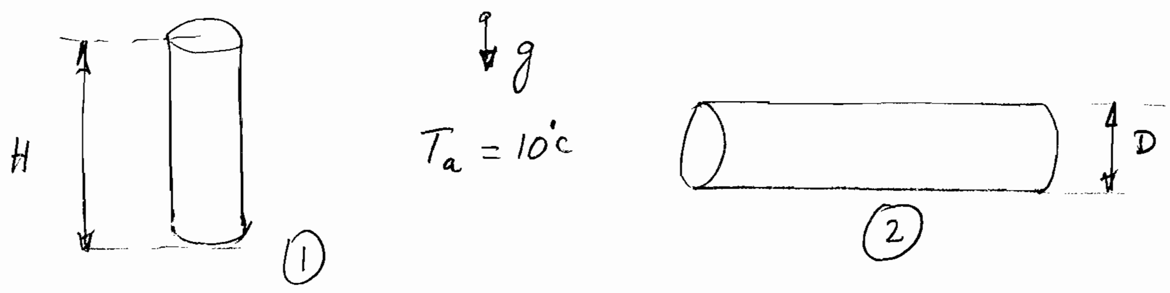
$\Rightarrow U = 34.22\ W/m^2\text{-}K$. (Smaller than the design value of 38)

This means that fouling has occurred.

To find ~~its~~ fouling factor:

$$U = \frac{1}{\left(\frac{1}{U_d} + R_f\right)} \Rightarrow R_f = 0.0029\ m^2\text{-}K/W$$

⑧



$H/D = 5$

Sol. From I^{st} law of thermodynamics, for beer

$$\frac{dE}{dt} = -\dot{Q}_{beer} = -hA(T_b - T_a)$$

$$mc\frac{dT_b}{dt} = hA(T_a - T_b)$$

$$-\frac{d\,\Delta T}{\Delta T} = \left(\frac{hA}{mc}\right)dt \Rightarrow \boxed{t \sim \left(\frac{mc}{hA}\right)}$$

only factor which depends on the orientation is h

$$\therefore \frac{t_1}{t_2} = \frac{h_2}{h_1}$$

$$T_b \overset{1/h_bA}{\text{—}} T_s \overset{1/h_aA}{\text{—}} T_a \Rightarrow \frac{1}{h} = \frac{1}{h_b} + \frac{1}{h_a}$$

from our scale analysis

$$h \sim \frac{1}{H}\left(k_f Ra_H^{1/4}\right) Pr^{1/4}$$

$$\frac{h_b}{h_a} = \frac{k_b\,(g\beta/\nu\alpha)_b^{1/4}\,Pr_b^{1/4}}{k_a\,(g\beta/\nu\alpha)_a^{1/4}\,Pr_a^{1/4}}$$

by treating beer as water

$$\frac{h_b}{h_a} = \frac{0.58 * (4910)^{1/4}}{0.025 * (125)^{1/4}} \left(\frac{6}{0.7}\right)^{1/4} = 99.37$$

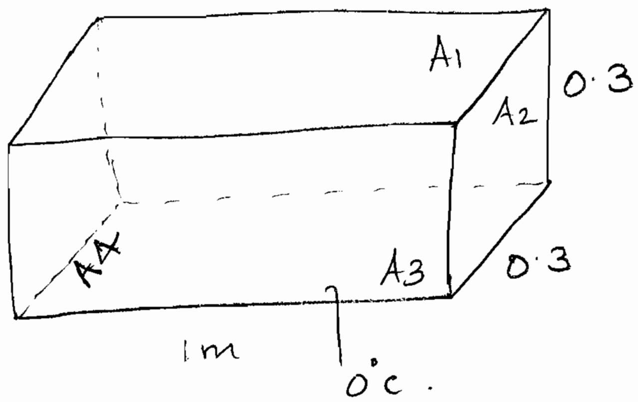
$$h \sim h_a \sim \frac{1}{H} R_a Ra_H^{1/4} Pr^{1/4} = \text{Const } H^{-1/4}$$

$$\therefore \frac{h_2}{h_1} = \frac{H_1^{1/4}}{H_2^{1/4}} = \left(\frac{H}{D}\right)^{1/4} = 5^{1/4} \approx 1.5$$

$$\therefore \boxed{\frac{t_1}{t_2} = 1.5}$$

Thus placing the beer bottle horizontally is prefferred as t_1 is 50% larger than t_2.

9



$T_a = 20°c$.

g

$h_{sf} = 333.4$ kJ/k

<u>Sol</u>: fluid properties @ $T_f = \left(\frac{0+20}{2}\right) = 10°c$.

$$\left(\frac{g\beta}{\alpha\nu}\right)_{air} \simeq 125 \quad cm^{-3}\,K^{-1}$$

$$k = 2.5\times10^{-4} \quad W/cm\text{-}K$$

$$\dot{q} = \dot{q}_1 A_1 + \dot{q}_4 A_4 + 2\dot{q}_2 A_2 + 2\dot{q}_3 A_3$$

$$= \left[h_1 A_1 + h_4 A_4 + 2h_2 A_2 + 2h_3 A_3\right]\Delta T$$

<u>for A_1</u>: Cold surface facing upward.

$$\overline{Nu}_L = 0.27\, Ra_L^{1/4} \qquad \text{(from Heat transfer book)}$$

$$L = \frac{0.3\times1}{2.6} = 0.1154\,m.$$

$$\therefore Ra_L = \frac{g\beta}{\alpha\nu}L^3\Delta T \simeq 125 * 0.1154^3 * 20\times10^6$$

$$Ra_L = 3.842\times10^6$$

$$\therefore h_1 = \frac{k}{L} * 0.27\left(3.842\times10^6\right)^{1/4} = 2.59 \quad W/m^2\text{-}K.$$

for A_4: Cold surface facing downward (from HT Book)

$$\overline{Nu_L} = 0.54\, Ra_L^{1/4} \quad \text{for} \quad Ra_L = 3.842 \times 10^6$$

$$= 23.9$$

$$\therefore h_4 = 5.18 \ \text{w/m}^2\text{K}$$

for A_2 & A_3. Both are vertical surface of $H = 0.3m$.

To see if the flow is laminar or turbulent

$$Gr_H = \frac{Ra_H}{Pr} = \left(\frac{g\beta}{\nu\alpha}\right) \frac{H^3 \Delta T}{Pr}$$

$$= 125 * 30^3 * \frac{20}{0.7} = 9.64 \times 10^7 < 10^9.$$

So the flow is laminar; $Ra_H = 6.75 \times 10^7$.

$$\overline{Nu_H} = \left(0.825 + 0.324\, Ra_H^{1/6}\right)^2 = \frac{\bar{h} H}{k}$$

$$= 54.21$$

$$\Rightarrow h_2 = h_3 = 4.52 \ \text{w/m}^2\text{-K}$$

$$\therefore \dot{q} = [2.59 * 0.3 + 5.18 * 0.3 + 2 * 4.52 * 0.09 + 2 * 0.3 * 4.52] * 20$$

$$\boxed{\dot{q} = 117.13 \ \text{W}}$$

$$\dot{m} h_{sf} = \dot{q} \Rightarrow \boxed{\dot{m} = 0.351 \ \text{g/sec}}$$