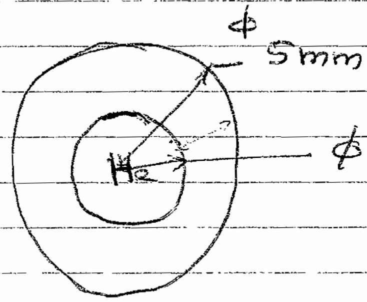
Example Sheet - 4

1)



Given: $D = 4\times10^{-14}$ $m^2/sec.$

$$\frac{\partial \rho_i}{\partial t} = -\nabla \cdot G_{diff}.$$

$$S = 4\times10^{-4} \ \frac{kmol}{m^3 bar} = 16\times10^{-4} \ \frac{kg}{m^3\text{-}bar}$$

i) Quasi-steady diffusion: $\left(\frac{\partial \rho_i}{\partial t} \approx 0\right)$

Spherically symmetric, with Fick's law:

$$\frac{1}{r^2}\frac{d}{dr}\left(\rho D r^2 \frac{dY_b}{dr}\right) = 0$$

Sol to this eqn. given

$$(A\,G_{diff}) = \rho\left(\frac{Y_{r_1} - Y_{r_2}}{R_m}\right) \qquad kg/sec.$$

$$\rho Y_{r_1} = S P_{r_1} = 16\times10^{-4} \quad kg/m^3$$

$$\rho Y_{r_2} = S P_{r_2} = 0$$ $\because$ P_{r_2} - Partial pressure of He in the surroundings is zero.

$$R_m = \frac{1}{4\pi D}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= \frac{1}{4*\pi*4\times10^{-14}}\left(\frac{1}{1\times10^{-3}} - \frac{1}{2.5\times10^{-3}}\right)$$

$$= 11.94\times10^{14} \quad sec/m^3$$

$\therefore$ mass flow rate $(A G_{diff}) = 1.34 \times 10^{-18}$ kg/sec //

$$\text{mass flux} = \frac{\text{mass flow rate}}{\text{Area}} = \frac{1.34 \times 10^{-18}}{1.26 \times 10^{-5}}$$

mass flux $= 1.06 \times 10^{-13}$ kg/m²-sec //

(ii) the mass balance for He

$$\frac{dm}{dt} = -(A G_{diff}) = -\frac{(S Y_{n})}{R_m}$$

But $m = \frac{pV}{RT}$ for ideal gas.

$$S Y_n = S P$$

$$\therefore \frac{V}{RT}\frac{dp}{dt} = -\frac{SP}{R_m} \Rightarrow \boxed{\frac{dp}{p} = -B\,dt. \quad B = \left(\frac{SRT}{R_m V}\right)}$$

S in $\frac{kg}{m^3 \cdot b}$

$$R = \frac{8.314 \times 10^3}{4} = 2.079 \times 10^3 \text{ J/kg-K}$$

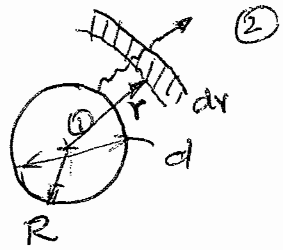
$T = 300$ K, $V = \frac{4}{3}\pi * (1 \times 10^{-3})^3 = 4.19 \times 10^{-9}\ m^3$

$$\therefore B = 1.99 \times 10^{-9} \text{ Sec}^{-1}.$$

$$\ln\left(\frac{p_t}{p_0}\right) = -Bt$$

$p_t = 0.5\, p_0 \Rightarrow \boxed{t = 11 \text{ yrs.}}$

2)



② → N_2 – which is the major constituent of the binary mixture.

$T = 675\,K$; $P = 1$ bar.

$$\rho = \frac{PM}{RT} = \frac{1\times10^5 * 28}{8314\times 675}$$

$$\rho = 0.499 \ kg/m^3.$$

$$G_1 = Y_1 G + G_{diff,1}$$

$$= Y_1 (G_1 + \cancel{G_2}^{\,0}) + G_{diff,1}$$

$$G_1 = -\frac{\rho D}{(1-Y_1)} \frac{dY_1}{dr}$$

But $\dot{M}_1 = G_1 * \text{Area} = -4\pi r^2 \dfrac{\rho D}{(1-Y_1)} \dfrac{dY_1}{dr}.$

Across the Control volume dr.

$\dfrac{d\dot{M}_1}{dr} = 0$ with $Y_1 \to Y_{1,\infty}$ as $r \to \infty$.

$Y_1 = Y_{1,s}$ @ $r = R$

$$\Rightarrow \quad 4\pi\rho D \frac{r^2}{1-Y_1} \frac{dY_1}{dr} = \text{Const.} = A.$$

$$\frac{dY_1}{1-Y_1} = \left(\frac{A}{4\pi\rho D}\right) \frac{dr}{r^2}$$

$$-\ln(1-Y_1) = -\left(\frac{A}{4\pi\rho D}\right)\frac{1}{r} - B_1$$

After some algebra $\dfrac{1-Y_1}{1-Y_{1,R}} = \dfrac{\exp\left[-\dfrac{A}{4\pi r\rho D}\right]}{\exp\left[-\dfrac{A}{4\pi R\rho D}\right]}$

Now as $r \to \infty$ $Y_1 \to Y_{1,\infty}$

$$\Rightarrow \quad A = 4\pi R \rho D \ln\left(\frac{1-Y_{1,\infty}}{1-Y_{1,R}}\right)$$

$$\boxed{A = 4\pi R \rho D \ln(1+B)} \quad \text{kg/sec.}$$

B — mass transfer number.

Rate of change of droplet mass.

$$\frac{dm}{dt} = -A$$

$$m = \rho_l V = \rho_l \frac{\pi d^{*3}}{6}$$

$$\Rightarrow \quad \frac{dd^*}{dt} = \frac{-4\rho D}{\rho_l d^*} \ln(1+B)$$

$$\frac{dd^{*2}}{dt} = -\frac{8\rho D}{\rho_l} \ln(1+B) \equiv -K$$

$$\int_0^t dt \quad \Rightarrow \quad \boxed{d^2 = d_o^2 - Kt.} \quad d^2\text{- law.}$$

<u>To get K.</u>

$\rho_l = 749 \ \text{kg/m}^3$

$\rho = \rho_{N_2}$ @ 675 K & 1 bar $= 0.499 \ \text{kg/m}^3 \approx 0.5 \ \text{kg/m}^3$

$$D = 8.1 \times 10^{-6} \left(\frac{675}{400}\right)^{3/2} = 1.78 \times 10^{-5} \ \text{m}^2/\text{sec.}$$

$$1+B = \left(\frac{1-Y_{1,\infty}}{1-Y_{1,R}}\right) = ? \qquad Y_{1,\infty} = 0.$$

To get $Y_{1,R}$:

Across the interface use Clausius - Clapeyron Equation :

$$\frac{dp}{P} = \frac{h_{fg}}{R} \frac{dT}{T^2}$$ because of phase change.

$$\Rightarrow \int_1^2 \Rightarrow \ln \frac{p_2}{p_1} = \frac{-h_{fg} M}{R} \left(\frac{1}{T_2} - \frac{1}{T_1} \right)$$

② Sat, vapor.

① Boiling.

$$\frac{P_{sat}}{1.0\text{ bar}} = \exp\left\{ - \frac{256 \times 170}{8.314} \left(\frac{1}{469.5} - \frac{1}{489.5} \right) \right.$$

$$\Rightarrow P_{sat} = 0.634 \text{ bar.}$$

$\Rightarrow$ $X_{1,R}$ - mole fraction of n- dodecane $= 0.634$.

But we require $Y_{1,R} = \frac{X_{1,R} M_1}{\bar{M}}$

$$\bar{M} = 0.634 * 170 + (1 - 0.634) * 28 = 118.03$$

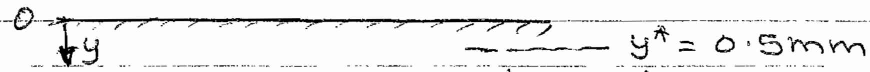
$$\therefore Y_{1,R} = 0.9132$$

$\therefore 1 + B = 11.52$ $\Rightarrow$ $K = 2.323 \times 10^{-7}\ m^2/\text{Sec.}$

$$\therefore t = \frac{d_0^2}{K} = \underline{0.173 \text{ Sec.}}$$

3)

C rich environment.



$D = 6\times10^{-10}$ m²/sec, $(\rho_c^{in}) = \rho_c^{in} = 16$ kg/m³

~~Sol~~: $\rho_{c,0} = 120$ kg/m³

$\rho_c = 50$ kg/m³ @ $y = 0.5$ mm; $t = ?$

Sol:

The diffusion is unsteady

$$\frac{\partial \rho_c}{\partial t} = -D\frac{\partial^2 \rho_c}{\partial y^2}$$

$$\Rightarrow \frac{\rho_c - \rho_{c,0}}{\rho_c^{in} - \rho_{c,0}} = erf\left(\frac{y}{2\sqrt{Dt}}\right)$$

∵ plate thicker is very large

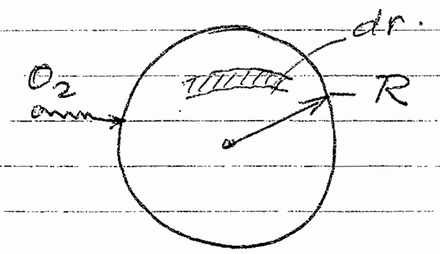
$$\frac{50 - 120}{16 - 120} = 0.673$$

From Tables of error function

$$erf^{-1}(0.673) = 0.694 = \frac{y^*}{2\sqrt{Dt}}$$

$$\therefore t = \left(\frac{y^*}{0.694}\right)^2 \frac{1}{4D} = \underline{\underline{3.6 \text{ min}}}.$$

4)



Respiration rate $\dot{\omega}^{*}_{O_2} = -k_1 C_{O_2}$

$\dot{\omega}_{O_2} = -k_1 \rho_{O_2}$ kg/m³ sec

Sol:

Balance of O_2 across the strip is

a)

$$\nabla \cdot G_{O_2} = \dot{\omega}_{O_2}$$

$$-\frac{1}{r^2}\frac{d}{dr}\left(r^2 D \frac{d\rho_{O_2}}{dr}\right) = -k_1 \rho_{O_2}$$

$$\Rightarrow \frac{D}{r^2}\frac{d}{dr}\left(r^2 \frac{d\rho_{O_2}}{dr}\right) = k_1 \rho_{O_2}$$

Let $\tau = r\rho_{O_2} \Rightarrow \frac{d\tau}{dr} = r\frac{d\rho_{O_2}}{dr} + \rho_{O_2}$

$$\Rightarrow \frac{d\rho_{O_2}}{dr} = \frac{1}{r}\frac{d\tau}{dr} - \frac{\tau}{r^2}$$

$$\Rightarrow \frac{D}{r^2}\frac{d}{dr}\left(r\frac{d\tau}{dr} - \tau\right) = k_1 \rho_{O_2}$$

$$\frac{D}{r^2}\left\{r\frac{d^2\tau}{dr^2} + \cancel{\frac{d\tau}{dr}} - \cancel{\frac{d\tau}{dr}}\right\} = k_1 \rho_{O_2}$$

$$\boxed{\frac{d^2\tau}{dr^2} - \frac{k_1}{D}\tau = 0.}$$

Two B.C.s: @ $r = R$; $\rho_{O_2} = \rho_{O_2,R} \Rightarrow \tau = R\rho_{O_2,R} = \tau_0$

@ $r = 0$, ρ_{O_2} should be finite.

solution to the above differential eqn.

$$\tau = C_1 \sinh\left(\frac{k_1}{D}\right)^{1/2} r + C_2 \cosh\left(\frac{k_1}{D}\right)^{1/2} r$$

$$\therefore \rho_{O_2} = \frac{C_1}{r} \sinh\left(\frac{k_1}{D}\right)^{1/2} r + \frac{C_2}{r} \cosh\left(\frac{k_1}{D}\right)^{1/2} r$$

@ $r = 0$ ρ_{O_2} is finite $\Rightarrow$ $C_2 = 0$.

@ $r = R$ $\rho_{O_{2,i}} = \rho_{O_{2,R}}$

$$C_1 = \frac{R\,\rho_{O_{2,R}}}{\sinh\left(\frac{k_1}{D}\right)^{1/2} R}$$

$$\therefore \boxed{\rho_{O_2} = \rho_{O_{2,R}} \left(\frac{R}{r}\right) \frac{\sinh\left[\left(\frac{k_1}{D}\right)^{1/2} r\right]}{\sinh\left[\left(\frac{k_1}{D}\right)^{1/2} R\right]}}$$

b) O_2 Consumption rate = diffusion rate @ the surface i.e. $r = R$.

$$\dot{R} = D(4\pi R^2) \left.\frac{d\rho_{O_2}}{dr}\right|_{r=R}$$

$$= 4\pi R^2 D\, \rho_{O_{2,R}} (\alpha \coth\alpha - 1)$$ //

with $\alpha = \left[\frac{k_1 R^2}{D}\right]^{1/2}$

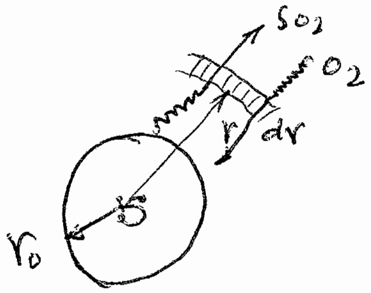
c) $R = 0.1$ mm; $D = 10^{-8}$ m²/sec. $k_1 = 20$ sec⁻¹

$\rho_{O_{2,R}} = 160 \times 10^{-5}$ kg/m³

$\rho_{O_2} (r \to 0) = 1.64 \times 10^{-4}$ kg/m³ //

$\dot{R} = 6.98 \times 10^{-14}$ kg/sec //

5)



on the Surface $S + O_2 \rightarrow SO_2$

$$\dot{\Omega}_s = - k_1' \rho_{O_2,s}$$

Air @ 1 bar, $T = 1200K$. $M = 28.84$

$$\rho = \frac{1\times10^5 \times 28.84}{8.314\times10^3 \times 1200} = 0.289 \ kg/m^3$$

$$\rho_{O_2,\infty} = \underbrace{x_{O_2,\infty}}_{0.21} \rho = 0.061 \ kg/m^3$$

Across the control volume

$\nabla \cdot G_{O_2} = 0$ Since there is no homogeneous reactions.

in spherically symmetric case

$$\frac{1}{r^2}\frac{d}{dr}\left(D r^2 \frac{d\rho_{O_2}}{dr}\right) = 0$$

$$r^2 \frac{d\rho_{O_2}}{dr} = C_1 \Rightarrow \rho_{O_2} = -\frac{C_1}{r} + C_2$$

as $r \rightarrow \infty$ $\rho_{O_2} \rightarrow \rho_{O_2,\infty}$

$$\boxed{\rho_{O_2} = -\frac{C_1}{r} + \rho_{O_2,\infty}}$$

On the surface:

$$D \left.\frac{d\rho_{O_2}}{dr}\right|_{r_0} = -\dot{\omega}_{O_2} = k_1' \rho_{O_2,s}$$

$$D \frac{c_1}{r_0^2} = k_1' \left(-\frac{c_1}{r_0} + \rho_{O_2,\infty} \right)$$

$$c_1 \left(\frac{D}{r_0^2} + \frac{k_1'}{r_0} \right) = k_1' \rho_{O_2,\infty}$$

$$\therefore \rho_{O_2} = -\frac{k_1' \rho_{O_2\infty}}{\left(D/r_0^2 + k_1'/r_0\right) r} + \rho_{O_2\infty}$$

$$\Rightarrow \boxed{\frac{\rho_{O_2}}{\rho_{O_2,\infty}} = 1 - \frac{1}{\left(D/k_1' r_0^2 + 1/r_0\right) r}}$$ finite rate.

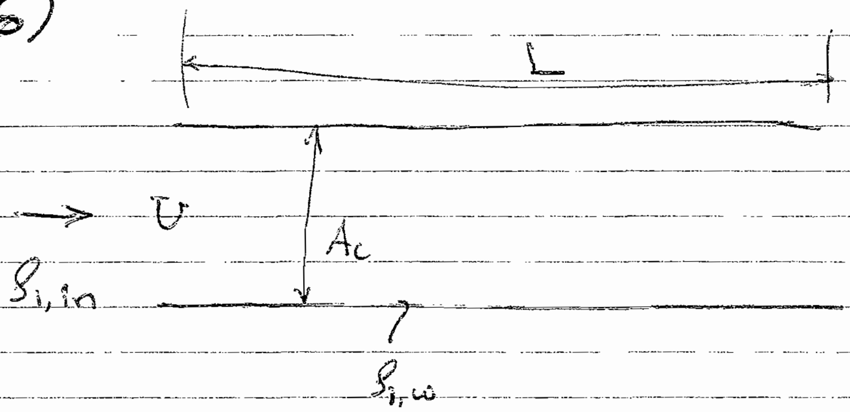
diffusion limited: $k_1' \longrightarrow \infty$

$$\Rightarrow \frac{\rho_{O_2}}{\rho_{O_2\infty}} = \left(1 - \frac{r_0}{r}\right) \Rightarrow \rho_{O_2,s} = 0.$$

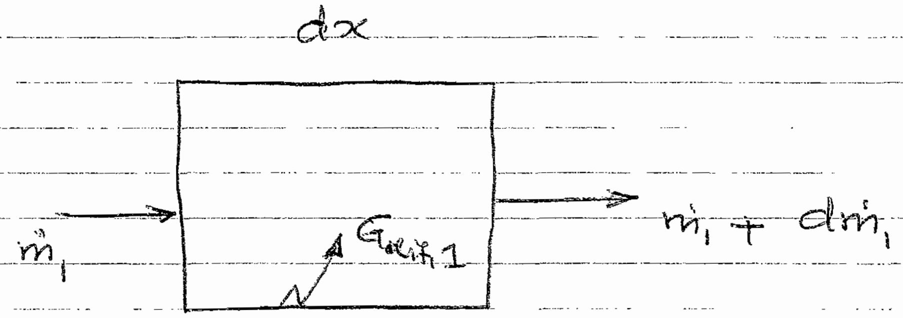
finite rate:

$$\frac{\rho_{O_2,s}}{\rho_{O_2,\infty}} = 1 - \frac{1}{\left(D/k_1' r_0 + 1\right)}$$

6)



Sol.



$$\dot{m}_1 = U A_c \rho_1 \Rightarrow d\dot{m}_1 = U A_c d\rho_1$$

But Conservation $\Rightarrow$ $d\dot{m}_1 = G_{diff,1}$

$$U A_c d\rho_1 = h_m P dx \left(\rho_{1w} - \rho_1\right)$$

$$\frac{d\rho_1}{(\rho_{1w} - \rho_1)} = \frac{h_m}{U A_c} P dx$$

Flow is fully developed, so h_m is const.

$$\Rightarrow -\ln\left(\rho_{1,w} - \rho_1\right) = \frac{h_m}{U A_c} P x + C_1$$

$x = 0$; $\rho = \rho_{1,in}$

$$\Rightarrow \frac{\rho_{1,w} - \rho_1}{\rho_{1,w} - \rho_{1,in}} = \exp\left[-\frac{h_m P}{U A_c} x\right]$$

Total mass transferred is

$$\dot{M}_1 = U A_c (\rho_{out,1} - \rho_{in,1})$$

From the above soln

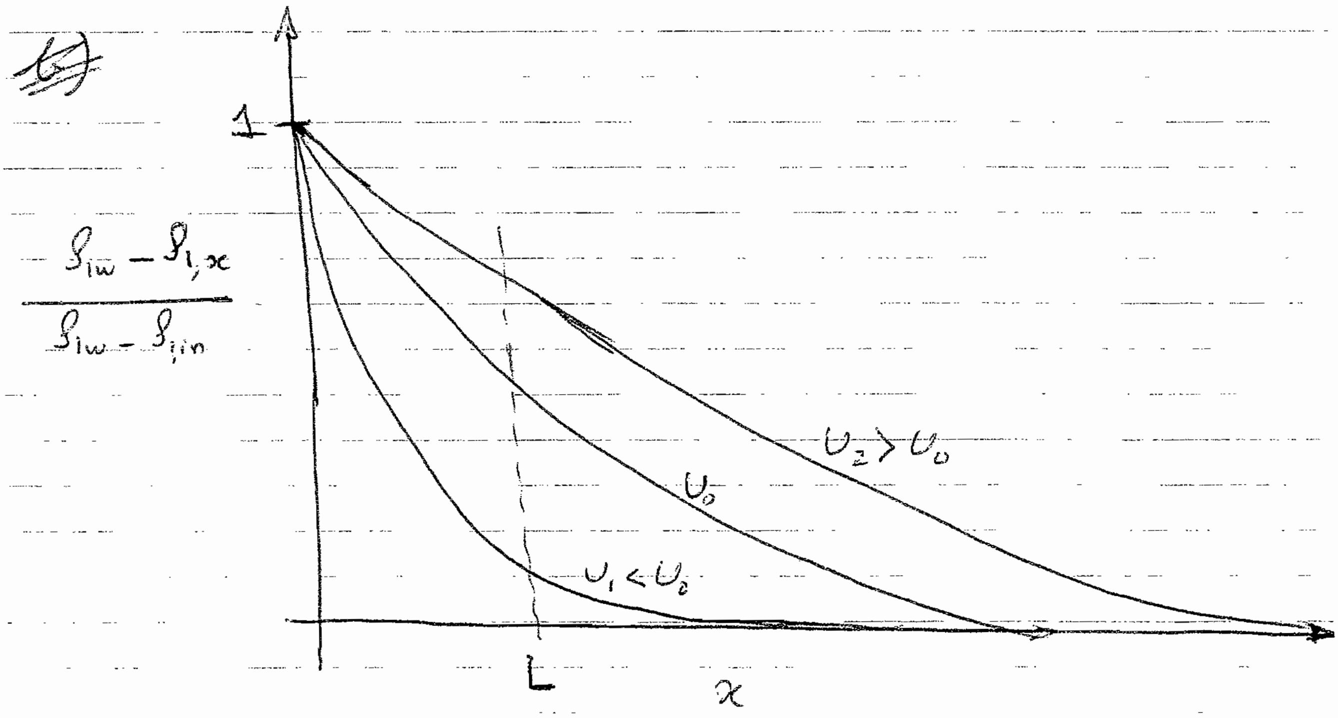
$$\frac{\rho_{w,1} - \rho_{out,1}}{\rho_{1,w} - \rho_{1,in}} = \exp\left[-\frac{h_m P L}{U A_c}\right]$$

$$\Rightarrow (\rho_{1,out} - \rho_{1,in}) = (\rho_{1,w} - \rho_{1,in})\left[1 - \exp\left(-\frac{h_m P L}{U A_c}\right)\right]$$

$$\therefore \boxed{\dot{M}_1 = U A_c (\rho_{1w} - \rho_{1,in})\left[1 - \exp\left(-\frac{h_m A}{U A_c}\right)\right]}$$

$A = PL$

b)



For given L,

ρ_{1L}, increases ~~with~~ as U decreases.

b) $U = 0.5$ m/sec $\quad d = 4\times10^{-2}$ m

$\nu = 2\times10^{-5}$ m²/sec, $\quad D = 2\times10^{-5}$ m²/sec

$$Re_d = \frac{Ud}{\nu} = \frac{0.5\times 4\times10^{-2}}{2\times10^{-5}} = 1000$$

laminar flow in circular tube

$$\Rightarrow Sh = 3.66 = \frac{h_m d}{D}$$

$$\Rightarrow h_m = \frac{3.66\times 2\times10^{-5}}{4\times10^{-2}} = 1.83\times10^{-3} \text{ m/sec}$$

$$\dot{M}_1 = UA_c(\rho_{1out} - \rho_{1in}) = 0.5 * 4\pi\times10^{-4}(0.0196 - 0.0128)$$

$$= 4.273\times10^{-6} \text{ kg/sec}$$

$$= 2\pi\times10^{-4}(0.0304 - 0.0128)\left[1 - \exp\left\{-\frac{h_m \pi d L\times 4}{0.5*\pi d^2}\right\}\right]$$

$$= 1.106\times10^{-5}\left[1 - \exp\left\{-\frac{\cancel{4}*1.83\times10^{-3}*L}{0.5*\cancel{4}\times10^{-2}}\right\}\right]$$

$$3.66\times10^{-1} L$$

$$\exp\{-3.66\times10^{-1} L\} = 1 - \frac{4.273\times10^{-6}}{1.106\times10^{-5}} = 0.6137$$

$$\therefore \boxed{L = 1.33 \text{ m}}$$