Engineering Tripos Part IIA **THIRD YEAR**

Module 3A5 : Thermodynamics and Power Generation

THERMODYNAMICS

Examples Paper 1

1. Air flows steadily with mass flowrate 2 kg/s through a duct of varying cross-sectional area. The static temperature of the flow is uniform at 400 K. At station 1 the static pressure is 5 bar and the flow velocity is 200 m/s. At station 2 the cross-sectional area is 50 % greater than at station 1 and the static pressure is 5.2 bar.

   Calculate the rate of production of entropy due to irreversibility between stations 1 and 2, and deduce formally whether the flow direction is from 1 to 2 or from 2 to 1.

2. Air flows steadily through various devices from state 1 (6 bar, 300 °C) to state 2 (4 bar, 100 °C). The air mass flowrate is 0.5 kg/s and the flow velocities at states 1 and 2 are low. The only heat exchange is with the environment at 25 °C.

   (a) Find the maximum shaft power that can be delivered between states 1 and 2.

   (b) Sketch a hypothetical ideal steady-flow process on a (T-s) diagram which would achieve this power output using ideal expanders and compressors only (not Carnot engines). Remember that all heat exchange with the environment must be carried out reversibly (*i.e.*, over a negligible temperature difference) and that ideal expanders and compressors can be isothermal as well as isentropic.

   (c) Prove analytically that the net shaft power output from the devices in the hypothetical process will indeed sum to the value obtained in part (a).

3. Air flows steadily at low velocity along a pipe in which there is a half-opened valve. At entry the air is at state 1 (6 bar, 300 °C) and at exit it is at state 2 (4 bar, 100 °C). The mass flowrate is 0.5 kg/s and there is heat loss from the pipe to the environment which is at 25 °C.

   (a) Calculate the actual shaft power, the ‘lost power’ due to heat transfer and the ‘lost power’ due to flow irreversibility.

   (b) Confirm that the sum of the actual and lost power terms equals the maximum power calculated in Q2.

   (c) Suggest a possible physical origin of the lost power due to flow irreversibility.

4. *A long question but worth doing carefully as you should then be able to tackle most exergy problems that don't involve mixtures of gases or chemical reactions.*

An open circuit gas turbine driving an electrical generator comprises an adiabatic compressor (which draws air directly from the atmosphere), a combustor, and an adiabatic turbine (which discharges into a duct leading to the exhaust stack). The flow in the duct and stack is adiabatic but not frictionless. For modelling purposes, the fuel injection can be neglected and the combustor replaced by a heater with zero pressure loss. The working fluid can be assumed to be the same perfect gas throughout and there are no bleeds for blade cooling or other purposes. Other relevant data are:

Compressor pressure ratio	=	15
Compressor isentropic efficiency	=	0.86
Heater outlet temperature	=	1400 K
Turbine isentropic efficiency	=	0.87
Pressure loss in outlet duct and stack	=	0.05 bar
Mass flowrate	=	25 kg/s
γ and c_p of the working fluid	=	1.36 and 1.10 kJ/kg K
Atmospheric (dead state) temperature	=	25 °C
Atmospheric (dead state) pressure	=	1.0 bar

It is convenient to set $h = s = 0$ at the dead state. Then $b_D = h_D - T_D s_D = 0$ and the exergy and availability functions are equal, $e = b$.

(a) Calculate values of h, s and $e = b = h - T_D s$ around the circuit.

(b) Calculate $\dot{Q}$, $\dot{E}_Q$, $\dot{W}_X$, $\dot{W}_{L,Q}$ and $\dot{W}_{L,CR}$ for each component in the circuit.

(c) Apply the energy equation to each component in turn, accounting for the change in fluid enthalpy flowrate between inlet and outlet.

(d) Apply the exergy equation to each component in turn, accounting for the change in fluid exergy flowrate between inlet and outlet.

(e) Draw up an overall energy balance showing how the heat supply rate is utilised. Calculate the thermal efficiency of the 'cycle'.

(f) Draw up an overall exergy balance showing how the exergy supply rate is utilised. Calculate the maximum possible thermal efficiency of the 'cycle' and the value of a suitably defined rational efficiency.

5. *Calculation of a GT combustor burning hydrogen. Because of the connection between CO_2 emission and global warming, there is interest in the possibility of firing GTs with hydrogen rather than carbon-based fuels like natural gas. The question gives practice in working with a molar- rather than a mass-based formulation.*

An adiabatic, steady-flow, gas turbine combustor is supplied with air (79% N_2 and 21% O_2 by volume) at 800 K from the compressor and, in a separate stream, hydrogen at 298.15 K. Both streams are at 25 bar pressure. After combustion (assumed to be complete) the products exit at 25 bar and 1800 K. The change in flow kinetic energy across the combustor may be neglected.

(a) Write the combustion equation in terms of the molar air/fuel ratio A.

(b) Using values of isobaric specific heat capacity c_p from the Data Book, calculate the molar isobaric heat capacities $\bar{c}_p$ of H_2, O_2, N_2 and H_2O at around ambient temperature. Comment on the values obtained.

(c) Write the SFEE in molar form and calculate the molar air-fuel ratio, A. For simplicity, assume that all gas species behave as perfect gases with constant molar heat capacity, $\bar{c}_p$ = 30.0 kJ/kmol K. For the reaction $H_2 + \frac{1}{2}O_2 \rightarrow H_2O$:

$$\Delta \overline{H}^0_{298} = -241.8 \text{ MJ/kmol } H_2$$

(d) Repeat the calculation of the molar air/fuel ratio using the molar enthalpy tables in the Data Book instead of assuming constant $\bar{c}_p$.

6. The chemical reaction in the combustor of Q5 is irreversible and is responsible for a large ‘lost power’ term in the GT plant.

(a) Calculate $\Delta \overline{S}^0_{298}$ for the reaction $H_2 + \frac{1}{2}O_2 \rightarrow H_2O$, given that,

$$\Delta \overline{G}^0_{298} = -228.6 \text{ MJ/kmol } H_2$$

(b) Use the steady-flow entropy equation to derive an expression for $\Delta \overline{S}_{\text{irrev}}$ the entropy created in the combustor per kmol of H_2 supplied. The expression should be written in terms of $\Delta \overline{S}^0_{298}$ and differences in molar entropies, for example terms like $[\bar{s}_{H2O}(p_2,T_2) - \bar{s}_{H2O}(p_0,T_0)]$. For each gas species, evaluate the molar entropy as if it existed at the combustor pressure of 25 bar rather than its individual partial pressure. This approach neglects the change in ‘entropy of mixing’ across the combustor but is in keeping with the definition of rational efficiency in terms of $\Delta \overline{G}^0_{298}$.

(c) Calculate $\Delta \overline{S}_{\text{irrev}}$. As in Q5(c), assume that all gas species behave as perfect gases with constant $\bar{c}_p$ = 30.0 kJ/kmol K. Take the molar air/fuel ratio from Q5(c) as A = 7.31.

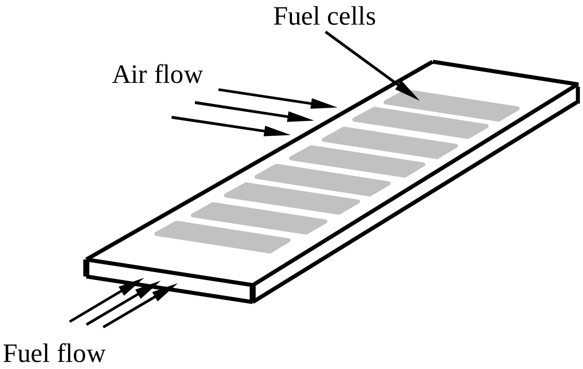
(d) If the combustion loss were the only loss in the power plant, what would be the rational efficiency of the plant?

7. The Rolls-Royce solid oxide fuel cell stack shown in the figure below is designed to operate steadily at a uniform temperature of 1200 K and a pressure of 5 bar. The fuel entering the fuel channels on the anode side is a methane-reformed mixture of H_2, H_2O and CO_2 with mole fractions 0.500, 0.375 and 0.125, respectively. At exit from the stack, 80% of the H_2 has been consumed. Air flows over the cathodes and supplies the O_2 for the electrochemical reaction. The air also serves to cool the stack and, because of its high flowrate, the mole fraction of O_2 remains close to 0.21 throughout the plant.

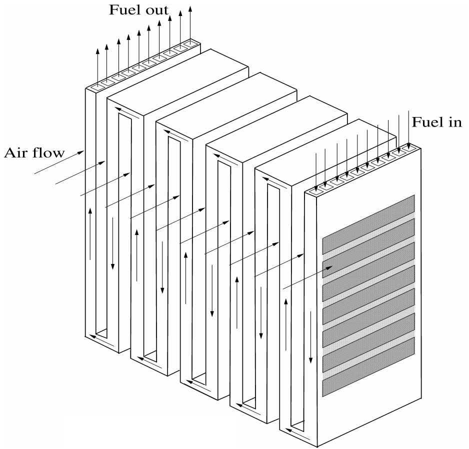
(a) Calculate the Gibbs potential at 1200 K and standard pressure (1 bar) for the reaction $H_2 + \frac{1}{2}O_2 \leftrightarrow H_2O$.

(b) Calculate the Nernst potential (at 1200 K and 5 bar) for a cell at stack inlet.

(c) Find the mole fractions of H_2, H_2O and CO_2 in the fuel channels at stack outlet and hence calculate the Nernst potential of a cell at stack outlet.



Module



Bundle

8. In the Rolls-Royce SOFC ‘module’ each individual three-layer fuel cell (anode–electrolyte–cathode) is screen-printed on the porous ceramic support structure and has dimensions 20×55 mm. On each side of a module there are 20 series-connected cells (only 8 are shown in the diagram on the previous page). Because of losses (mainly ohmic), the voltage generated by each cell is only 85% of the average Nernst potential in the stack (as calculated in Q7). In order to prevent degradation of the cell materials, the electric current density in each cell is limited to 3000 A/m^2.

10 modules are series-connected to make a ‘bundle’ as shown in the figure. Bundles are then assembled in parallel to provide the required power output from the ‘stack’. Rolls-Royce plans to develop a 250 kW stack as the basic unit.

(a) Calculate the electric power output from a module, the voltage generated by a bundle and the number of bundles required for a 250 kW stack. If the overall dimensions of a module are about 500×65×6 mm and the adjacent air gap is about 6 mm, make a rough estimate of the volume of a 250 kW stack.

(b) For a 250 kW stack, calculate the rate of consumption of H_2 in kmol/s.

(c) The primary fuel supplied to the plant is methane and all the H_2 is generated internally via the steam reforming reaction, $CH_4 + 2H_2O \rightarrow 4H_2 + CO_2$. Assume that this reaction goes to completion so there is no CO present at stack inlet and take the stack hydrogen utilisation as 80% (as in Q7). Calculate the rate of supply of CH_4 to the plant in kmol/s.

(d) The methane is supplied to the plant at 25 °C and 1 bar. For the oxidation reaction, $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$, $\Delta\overline{G}^0_{298} = -800.0$ MJ/kmol of CH_4. Assuming that all the power generated comes from the fuel cell stack (*i.e.*, ignoring the contribution from any associated turbomachinery), calculate the rational efficiency of the overall plant.

Tripos questions for revision :

2006 (3A5)	Q1 (a)
2005 (3A5 Sample Paper)	Q1
2005 (3A5 Old style)	Q1 (b)
1998 (G10)	Q1

ANSWERS

1. 36.4 W/K

2. (a) 53.6 kW

3 (a) 0 kW, 36.2 kW, 17.4 kW

4. Values of *e* around circuit : 0.0, 373.4, 939.9, 221.9, 217.7 kJ/kg

Inlet exergy flowrate	0.000 MW
Compressor power	–9.991 MW
Compressor lost power	0.656 MW
Turbine power	16.927 MW
Turbine lost power	1.023 MW
Outlet duct lost power	0.106 MW
Exhaust exergy flowrate	5.441 MW

Thermal efficiency = 34.2 % Rational efficiency = 49.0 %

5. (a) $H_2 + A(0.21O_2 + 0.79N_2) \rightarrow H_2O + (0.21A - 0.5)O_2 + (0.79A)N_2$
   (c) 7.31
   (d) 5.97

6. (a) – 44.27 kJ/kmol K
   (b)
$$\Delta\bar{S}_{\text{irrev}} = \Delta\bar{S}^0_{298} + [\bar{s}_{H2O}(p,T_2) - \bar{s}_{H2O}(p_0,T_0)] - 0.5[\bar{s}_{O2}(p,T_2) - \bar{s}_{O2}(p_0,T_0)] - [\bar{s}_{H2}(p,T_0) - \bar{s}_{H2}(p_0,T_0)] + (0.21A)[\bar{s}_{O2}(p,T_2) - \bar{s}_{O2}(p,T_1)] + (0.79A)[\bar{s}_{N2}(p,T_2) - \bar{s}_{N2}(p,T_1)]$$
   (c) 173.9 kJ/K per kmol H_2 supplied. (Taking A = 7.31)
   (d) 77.3 %

7. (a) 0.940 volts
   (b) 0.956 volts
   (c) $X_{H2} = 0.100$, $X_{H2O} = 0.775$, $X_{CO2} = 0.125$, 0.836 volts

8. (a) 100 watts, 152 volts, 250 bundles, 1 m^3
   (b) 1.71×10^{-3} kmol/s
   (c) 5.34×10^{-4} kmol/s
   (d) 59 %.

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