

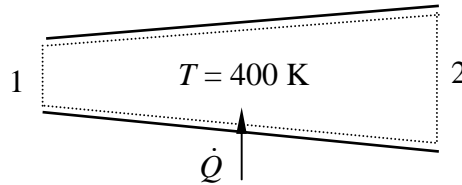
Module 3A5 : Thermodynamics and Power Generation

THERMODYNAMICS

Solutions to Examples Paper 1

(J.B. Young, October 2007)

1.



Work as if the flow is from 1 to 2 :

$$\text{At station 1, } p_1 = 5.0 \text{ bar, } T_1 = 400 \text{ K} \rightarrow \rho_1 = \frac{p_1}{RT_1} = \frac{5.0 \times 10^5}{287.0 \times 400.0} = 4.355 \text{ kg/m}^3$$

$$\text{At station 2, } p_2 = 5.2 \text{ bar, } T_2 = 400 \text{ K} \rightarrow \rho_2 = \frac{p_2}{RT_2} = \frac{5.2 \times 10^5}{287.0 \times 400.0} = 4.530 \text{ kg/m}^3$$

$$V_1 = 200 \text{ m/s and } A_2/A_1 = 1.5 \rightarrow V_2 = \frac{\rho_1}{\rho_2} \frac{A_1}{A_2} V_1 = \frac{4.355}{4.530} \times \frac{1}{1.5} \times 200.0 = 128.18 \text{ m/s}$$

SFEE for control volume with zero shaft work and $h_2 = h_1$ because $T_2 = T_1$:

$$\dot{Q} = \dot{m} \left[(h_2 - h_1) + \frac{(V_2^2 - V_1^2)}{2} \right] = 2 \times \left(\frac{128.18^2 - 200.0^2}{2} \right) = -23570.0 \text{ W}$$

Entropy equation for control volume :

$$\begin{aligned} \Delta \dot{S}_{\text{irrev}} &= \dot{m}(s_2 - s_1) - \int_1^2 \frac{d\dot{Q}}{T} = -\dot{m} R \ln \left(\frac{p_2}{p_1} \right) - \frac{\dot{Q}}{T} \\ &= -2.0 \times 287.0 \times \ln \left(\frac{5.2}{5.0} \right) + \frac{23570.0}{400.0} = -22.51 + 58.93 \\ &= 36.42 \text{ W/K} \end{aligned}$$

As $\Delta \dot{S}_{\text{irrev}} \geq 0$ the flow must be from 1 to 2.

Note that the entropy flowrate ($\dot{m}s$) is decreased by the cooling heat transfer but increased by the entropy production due to irreversibility. The net effect is a decrease in entropy flowrate. Also note that the pressure rises in the flow direction because of the area increase. Fluids do not necessarily flow from high to low pressure.

2(a) Maximum shaft power from a steady-flow process from state 1 to state 2 is,

$$[\dot{W}_X]_{\text{MAX}} = \dot{m}(b_1 - b_2) = \dot{m}[(h_1 - T_0 s_1) - (h_2 - T_0 s_2)]$$

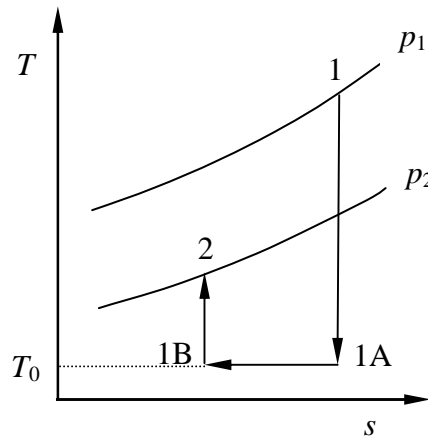
Hence, for a perfect gas,

$$[\dot{W}_X]_{\text{MAX}} = \dot{m} \left\{ c_p (T_1 - T_2) - T_0 \left[c_p \ln \left(\frac{T_1}{T_2} \right) - R \ln \left(\frac{p_1}{p_2} \right) \right] \right\}$$

From the Data Book, for air, $R = 0.287$, $c_p = 1.005$ kJ/kg K.

$$\begin{aligned} [\dot{W}_X]_{\text{MAX}} &= 0.5 \left\{ 1.005(573.15 - 373.15) - 298.15 \left[1.005 \ln \left(\frac{573.15}{373.15} \right) - 0.287 \ln \left(\frac{6}{4} \right) \right] \right\} \\ &= 0.5 (201.00 - 93.90) = 53.55 \text{ kW} \end{aligned}$$

(b) Conceptually, the maximum power could be achieved as shown in the (T - s) diagram below. This would require a reversible adiabatic (isentropic) expander ($1 \rightarrow 1A$), a reversible isothermal compressor operating at T_0 ($1A \rightarrow 1B$), and a reversible adiabatic (isentropic) compressor ($1B \rightarrow 2$). All heat transfer then occurs reversibly at T_0 .



(c) From the SFEE, $1 \rightarrow 1A$: $[\dot{W}_X]_{1-1A} = \dot{m}(h_1 - h_{1A})$

From the SFEE, $1A \rightarrow 1B$: $[\dot{Q}]_{1A-1B} - [\dot{W}_X]_{1A-1B} = \dot{m}(h_{1B} - h_{1A})$

From the SFSE, $1A \rightarrow 1B$: $[\dot{Q}]_{1A-1B} = \dot{m}T_0(s_{1B} - s_{1A})$

Hence : $[\dot{W}_X]_{1A-1B} = \dot{m}(h_{1A} - h_{1B}) - \dot{m}T_0(s_{1A} - s_{1B})$

From the SFEE, $1B \rightarrow 2$: $[\dot{W}_X]_{1B-2} = \dot{m}(h_{1B} - h_2)$

$[\dot{W}_X]_{1-1A} + [\dot{W}_X]_{1A-1B} + [\dot{W}_X]_{1B-2} = \dot{m}[(h_1 - T_0 s_1) - (h_2 - T_0 s_2)] = \dot{m}(b_1 - b_2)$

Note that we did not have to assume that the fluid was a perfect gas.

3(a) For the flow in the pipe the actual shaft power is zero, $\dot{W}_X = 0$.

The lost power associated with the heat transfer is found from :

$$\dot{W}_{L,Q} = \int_1^2 \left(1 - \frac{T_0}{T}\right) d\dot{Q}$$

This is easy to remember because the integrand is the power output that would be obtained from a Carnot cycle operating between local flow temperature T and reservoir temperature T_0 with heat transfer rate $d\dot{Q}$ from the flow. Because the temperature is changing along the pipe, an integration is required. $d\dot{Q}$ can be related to the differential temperature change along the pipe dT by the SFEE :

$$d\dot{Q} = -\dot{m}c_p dT$$

Hence :

$$\begin{aligned} \dot{W}_{L,Q} &= -\int_1^2 \left(1 - \frac{T_0}{T}\right) \dot{m}c_p dT = -\dot{m} \left[c_p(T_2 - T_1) - c_p T_0 \ln\left(\frac{T_2}{T_1}\right) \right] \\ &= -0.5 \times 1.005 \left[(373.15 - 573.15) - 298.15 \ln\left(\frac{373.15}{573.15}\right) \right] \\ &= 36.20 \text{ kW} \end{aligned}$$

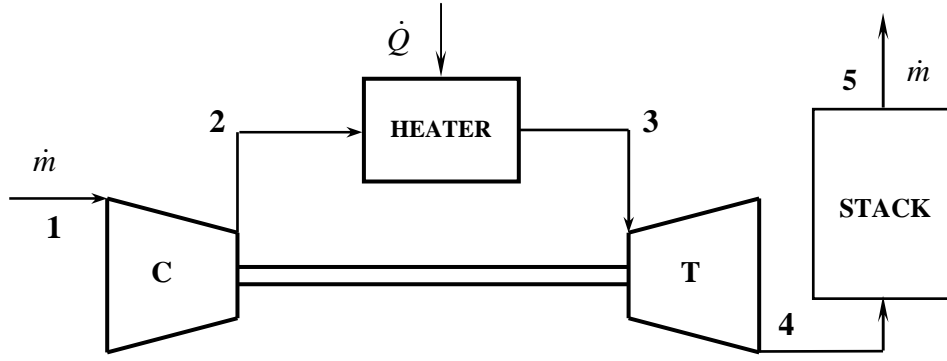
The lost power due to flow irreversibility is obtained from the SFSE :

$$\begin{aligned} \dot{W}_{L,CR} &= T_0 \Delta \dot{S}_{\text{irrev}} = \dot{m}T_0 (s_2 - s_1) - \int_1^2 \left(\frac{T_0}{T}\right) d\dot{Q} \\ &= \dot{m} \left[c_p T_0 \ln\left(\frac{T_2}{T_1}\right) - RT_0 \ln\left(\frac{p_2}{p_1}\right) \right] - \dot{m}c_p T_0 \ln\left(\frac{T_2}{T_1}\right) \\ &= -\dot{m}RT_0 \ln\left(\frac{p_2}{p_1}\right) \\ &= -0.5 \times 0.287 \times 298.15 \times \ln\left(\frac{4}{6}\right) = 17.35 \text{ kW} \end{aligned}$$

(b) $\dot{W}_X + \dot{W}_{L,Q} + \dot{W}_{L,CR} = 0 + 36.20 + 17.35 = 53.55 \text{ kW} = [\dot{W}_X]_{\text{MAX}}$ from Q2.

(c) The lost power due to flow irreversibility is related to the pressure loss in the pipe. This will be caused by turbulent (and ultimately viscous) dissipation of flow kinetic energy (particularly after the half open valve).

4.



Gas Data : $\gamma = 1.36$, $c_p = 1.10$ kJ/kg K, $R = c_p(\gamma-1)/\gamma = 0.2912$ kJ/kg K

For convenience, set $h_D = s_D = 0$. Hence $b_D = h_D - T_D s_D = 0$.

(a) Calculate values of h , s and e around the circuit :

Intake : $T_1 = 298.15$ K, $p_1 = 1$ bar. Hence, $h_1 = s_1 = b_1 = 0$. Hence, $e_1 = b_1 - b_D = 0$

Compressor exit: $p_2 = 15$ bar, $\eta_C = 0.86$

$$T_{2S} = T_1 (p_2/p_1)^{(\gamma-1)/\gamma} = 610.59 \text{ K}$$

$$T_2 = T_1 + (T_{2S} - T_1)/\eta_C = 661.46 \text{ K}$$

$$h_2 = h_1 + c_p (T_2 - T_1) = 399.64 \text{ kJ/kg}$$

$$s_2 = s_1 + c_p \ln(T_2/T_1) - R \ln(p_2/p_1) = 0.087954 \text{ kJ/kg K}$$

$$e_2 = b_2 - b_D = h_2 - T_D s_2 = 373.42 \text{ kJ/kg}$$

Heater exit: $T_3 = 1400$ K, $p_3 = 15$ bar

$$h_3 = h_1 + c_p (T_3 - T_1) = 1212.04 \text{ kJ/kg}$$

$$s_3 = s_1 + c_p \ln(T_3/T_1) - R \ln(p_3/p_1) = 0.91271 \text{ kJ/kg K}$$

$$e_3 = b_3 - b_D = h_3 - T_D s_3 = 939.92 \text{ kJ/kg}$$

Turbine exit: $p_4 = 1.05$ bar, $\eta_T = 0.87$

$$T_{4S} = T_3 (p_4/p_3)^{(\gamma-1)/\gamma} = 692.49 \text{ K}$$

$$T_4 = T_3 - \eta_T (T_3 - T_{4S}) = 784.47 \text{ K}$$

$$h_4 = h_1 + c_p (T_4 - T_1) = 534.96 \text{ kJ/kg}$$

$$s_4 = s_1 + c_p \ln(T_4/T_1) - R \ln(p_4/p_1) = 1.04995 \text{ kJ/kg K}$$

$$e_4 = b_4 - b_D = h_4 - T_D s_4 = 221.92 \text{ kJ/kg}$$

Stack exit : $p_5 = 1.0$ bar

$$T_5 = T_4 = 784.47 \text{ K}$$

$$h_5 = h_4 \text{ (SFEE)} = 534.96 \text{ kJ/kg}$$

$$s_5 = s_1 + c_p \ln(T_5/T_1) - R \ln(p_5/p_1) = 1.06415 \text{ kJ/kg K}$$

$$e_5 = b_5 - b_D = h_5 - T_D s_5 = 217.68 \text{ kJ/kg}$$

Station	p (bar)	T (K)	h (kJ/kg)	s (kJ/kg K)	e (kJ/kg)
(1) Intake	1.00	298.15	0.0	0.0	0.0
(2) Compressor exit	15.00	661.46	399.64	0.08795	373.42
(3) Heater exit	15.00	1400.00	1212.04	0.91271	939.92
(4) Turbine exit	1.05	784.47	534.96	1.04995	221.92
(5) Stack	1.00	784.47	534.96	1.06415	217.68

(b) Calculate the heat rates, power and lost power terms (with $\dot{m} = 25$ kg/s) :

Compressor :

$$\begin{aligned}\dot{Q}, \dot{E}_Q, \dot{W}_{L,Q} &= 0 \\ \dot{W}_C = \dot{m}(h_1 - h_2) &= -9.991 \text{ MW} \\ \dot{W}_{L,CR} = \dot{m}T_0(s_2 - s_1) &= 0.656 \text{ MW}\end{aligned}$$

Heater :

$$\begin{aligned}\dot{W}_{L,Q}, \dot{W} &= 0 \\ \dot{Q}_{IN} = \dot{m}(h_3 - h_2) &= 20.310 \text{ MW} \\ \dot{E}_Q = \dot{m}[c_p(T_3 - T_2) - c_p T_0 \ln(T_3/T_2)] &= 14.162 \text{ MW} \\ \dot{W}_{L,CR} = \dot{m}T_0[(s_3 - s_2) - c_p \ln(T_3/T_2)] &= 0\end{aligned}$$

Turbine :

$$\begin{aligned}\dot{Q}, \dot{E}_Q, \dot{W}_{L,Q} &= 0 \\ \dot{W}_T = \dot{m}(h_3 - h_4) &= 16.927 \text{ MW} \\ \dot{W}_{L,CR} = \dot{m}T_0(s_4 - s_3) &= 1.023 \text{ MW}\end{aligned}$$

Outlet duct :

$$\begin{aligned}\dot{Q}, \dot{E}_Q, \dot{W}_{L,Q}, \dot{W} &= 0 \\ \dot{W}_{L,CR} = \dot{m}T_0(s_5 - s_4) &= 0.106 \text{ MW}\end{aligned}$$

(c) Energy balance for each component :

Compressor	$\dot{m}h_2 = \dot{m}h_1 - \dot{W}_C$	9.991 MW	=	0.0 - (-9.991)
Heater	$\dot{m}h_3 = \dot{m}h_2 + \dot{Q}_{IN}$	30.301 MW	=	9.991 + 20.310
Turbine	$\dot{m}h_4 = \dot{m}h_3 - \dot{W}_T$	13.374 MW	=	30.301 - 16.927
Outlet duct	$\dot{m}h_5 = \dot{m}h_4$	13.374 MW	=	13.374

(d) Exergy balance for each component :

Compressor	$\dot{m}e_2 = \dot{m}e_1 - \dot{W}_C - \dot{W}_{L,CR}$	9.335 MW	=	0.0 - (-9.991) - 0.656
Heater	$\dot{m}e_3 = \dot{m}e_2 + \dot{E}_Q$	23.497 MW	=	9.335 + 14.162
Turbine	$\dot{m}e_4 = \dot{m}e_3 - \dot{W}_T - \dot{W}_{L,CR}$	5.547 MW	=	23.497 - 16.927 - 1.023
Outlet duct	$\dot{m}e_5 = \dot{m}e_4 - \dot{W}_{L,CR}$	5.441 MW	=	5.547 - 0.106

(e) Overall energy balance :

Rate of heat input	20.310 MW	100.00 %	
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Inlet enthalpy flowrate	0.000 MW	0.00 %	
Net power output	6.936 MW	34.15 %	← Thermal efficiency
Exhaust enthalpy flowrate	13.374 MW	65.85 %	
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Total	20.310 MW	100.00 %	
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(f) Overall exergy Balance :

Rate of exergy supply	14.162 MW	100.00 %	
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Inlet exergy flowrate	0.000 MW	0.00 %	
Net power output	6.936 MW	48.98 %	← Rational efficiency
Compressor lost power	0.656 MW	4.63 %	
Turbine lost power	1.023 MW	7.22 %	
Outlet duct lost power	0.106 MW	0.75 %	
Exhaust exergy flowrate	5.441 MW	38.42 %	← Huge loss
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Total	14.162 MW	100.00 %	
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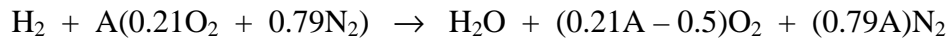
$$\text{Maximum thermal efficiency} = \frac{\dot{E}_Q}{\dot{Q}_{IN}} = \frac{14.162}{20.310} \equiv 69.73 \%$$

The rational efficiency is poor because of the very large exhaust exergy flowrate.

5.



(a) Let the molar air fuel ratio be A. For complete combustion the chemical equation is :



(b) From the table of perfect gas properties (M is the molar mass in kg/kmol) :

$$\text{Hydrogen (H}_2\text{)} \quad \bar{c}_p = M c_p = 2 \times 14.20 = 28.4 \text{ kJ/kmol K}$$

$$\text{Oxygen (O}_2\text{)} \quad \bar{c}_p = M c_p = 32 \times 0.92 = 29.4 \text{ kJ/kmol K}$$

$$\text{Nitrogen (N}_2\text{)} \quad \bar{c}_p = M c_p = 28 \times 1.04 = 29.1 \text{ kJ/kmol K}$$

These are similar because H_2 , O_2 and N_2 are all diatomic near-perfect gases. The heat capacities increase at high temperature because vibrational states of the atom pairs become activated. From the table of transport properties of saturated water and steam:

$$\text{Water vapour (H}_2\text{O)} \quad \bar{c}_p = M c_p = 18 \times 1.87 = 33.7 \text{ kJ/kmol K}$$

The molar heat capacity of H_2O is higher because the molecule is polyatomic and can store energy in different rotational and vibrational forms. Nevertheless, the difference is not huge (at ambient temperature).

(c) The process is adiabatic so application of the SFEE in molar form gives,

$$\begin{aligned} \bar{h}_{\text{H}_2}(T_0) + (0.21\text{A})\bar{h}_{\text{O}_2}(T_1) + (0.79\text{A})\bar{h}_{\text{N}_2}(T_1) \\ = \bar{h}_{\text{H}_2\text{O}}(T_2) + (0.21\text{A} - 0.5)\bar{h}_{\text{O}_2}(T_2) + (0.79\text{A})\bar{h}_{\text{N}_2}(T_2) \end{aligned}$$

By definition,

$$\Delta \bar{H}_{298}^0 = \bar{h}_{\text{H}_2\text{O}}(T_0) - \bar{h}_{\text{H}_2}(T_0) - 0.5\bar{h}_{\text{O}_2}(T_0)$$

Substituting this expression and rearranging gives,

$$\begin{aligned} -\Delta \bar{H}_{298}^0 = [\bar{h}_{\text{H}_2\text{O}}(T_2) - \bar{h}_{\text{H}_2\text{O}}(T_0)] - 0.5[\bar{h}_{\text{O}_2}(T_2) - \bar{h}_{\text{O}_2}(T_0)] + \\ (0.21\text{A})[\bar{h}_{\text{O}_2}(T_2) - \bar{h}_{\text{O}_2}(T_1)] + (0.79\text{A})[\bar{h}_{\text{N}_2}(T_2) - \bar{h}_{\text{N}_2}(T_1)] \end{aligned}$$

Each term in square brackets on the RHS involves a difference in molar enthalpy.

Assuming perfect gas behaviour with all species having the same \bar{c}_p ,

$$-\Delta\bar{H}_{298}^0 = \bar{c}_p(T_2 - T_0) - 0.5\bar{c}_p(T_2 - T_0) + (0.21A)\bar{c}_p(T_2 - T_1) + (0.79A)\bar{c}_p(T_2 - T_1)$$

Inserting the data :

$$241.8 \times 10^3 = (1.0 - 0.5) \times 30.0 \times (1800.0 - 298.15) + \\ (0.21 + 0.79) \times A \times 30.0 \times (1800.0 - 800.0)$$

Hence, the molar air-fuel ratio $A = 7.309$

(d) Using the molar enthalpy tables in the Data Book :

$$\bar{h}_{\text{H}_2\text{O}}(T_2) - \bar{h}_{\text{H}_2\text{O}}(T_0) = 72.58 - 9.90 = 62.68 \text{ MJ/kmol}$$

$$\bar{h}_{\text{O}_2}(T_2) - \bar{h}_{\text{O}_2}(T_0) = 60.35 - 8.66 = 51.69 \text{ MJ/kmol}$$

$$\bar{h}_{\text{O}_2}(T_2) - \bar{h}_{\text{O}_2}(T_1) = 60.35 - 24.50 = 35.85 \text{ MJ/kmol}$$

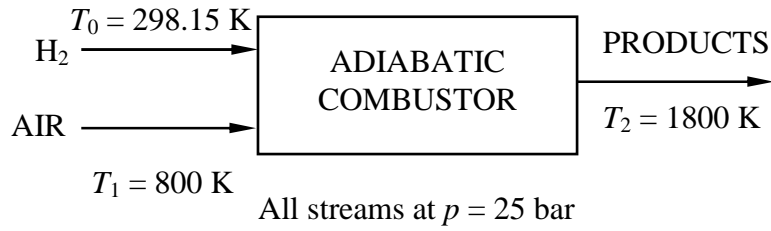
$$\bar{h}_{\text{N}_2}(T_2) - \bar{h}_{\text{N}_2}(T_1) = 57.67 - 23.72 = 33.95 \text{ MJ/kmol}$$

Inserting the data into the SFEE :

$$241.8 = 62.68 - 0.5 \times 51.69 + 0.21 \times A \times 35.85 + 0.79 \times A \times 33.95$$

Hence, the molar air-fuel ratio $A = 5.967$ (This, of course, is an accurate value)

7.



- (a) To calculate the standard entropy change of reaction at 25 °C :

$$\Delta \bar{S}_{298}^0 = \frac{\Delta \bar{H}_{298}^0 - \Delta \bar{G}_{298}^0}{T_0} = \frac{(-241.8 \times 10^3) - (-228.6 \times 10^3)}{298.15} = -44.27 \text{ kJ/kmol K}$$

- (b) The combustor is adiabatic so there is no entropy change across it due to heat transfer. Thus, the entropy production rate due to the irreversible chemical reaction equals the entropy flowrate leaving minus the entropy flowrate entering. We are told to evaluate all molar entropies at pressure $p = 25 \text{ bar}$ (*i.e.*, ignore the change in 'entropy of mixing' across the combustor). Hence, per kmol H_2 supplied,

$$\Delta \bar{S}_{\text{irrev}} = [\bar{s}_{\text{H}_2\text{O}}(p, T_2) + (0.21A - 0.5)\bar{s}_{\text{O}_2}(p, T_2) + (0.79A)\bar{s}_{\text{N}_2}(p, T_2)] - [\bar{s}_{\text{H}_2}(p, T_0) + (0.21A)\bar{s}_{\text{O}_2}(p, T_1) + (0.79A)\bar{s}_{\text{N}_2}(p, T_1)]$$

By definition,

$$\Delta \bar{S}_{298}^0 = \bar{s}_{\text{H}_2\text{O}}(p_0, T_0) - \bar{s}_{\text{H}_2}(p_0, T_0) - 0.5\bar{s}_{\text{O}_2}(p_0, T_0)$$

Substituting into the expression for $\Delta \bar{S}_{\text{irrev}}$ and rearranging gives,

$$\begin{aligned} \Delta \bar{S}_{\text{irrev}} = & \Delta \bar{S}_{298}^0 + [\bar{s}_{\text{H}_2\text{O}}(p, T_2) - \bar{s}_{\text{H}_2\text{O}}(p_0, T_0)] - \\ & 0.5[\bar{s}_{\text{O}_2}(p, T_2) - \bar{s}_{\text{O}_2}(p_0, T_0)] - [\bar{s}_{\text{H}_2}(p, T_0) - \bar{s}_{\text{H}_2}(p_0, T_0)] + \\ & (0.21A)[\bar{s}_{\text{O}_2}(p, T_2) - \bar{s}_{\text{O}_2}(p, T_1)] + (0.79A)[\bar{s}_{\text{N}_2}(p, T_2) - \bar{s}_{\text{N}_2}(p, T_1)] \end{aligned}$$

Each term in square brackets on the RHS involves a difference in molar entropy.

- (c) Take $\bar{R} = 8.3145 \text{ kJ/kmol K}$ and $\bar{c}_p = 30.0 \text{ kJ/kmol K}$ for all gas species. Hence :

$$[\bar{s}_{\text{H}_2\text{O}}(p, T_2) - \bar{s}_{\text{H}_2\text{O}}(p_0, T_0)] = \bar{c}_p \ln\left(\frac{T_2}{T_0}\right) - \bar{R} \ln\left(\frac{p}{p_0}\right) = 27.175 \text{ kJ/kmol K}$$

$$[\bar{s}_{\text{O}_2}(p, T_2) - \bar{s}_{\text{O}_2}(p_0, T_0)] = \bar{c}_p \ln\left(\frac{T_2}{T_0}\right) - \bar{R} \ln\left(\frac{p}{p_0}\right) = 27.175 \text{ kJ/kmol K}$$

$$[\bar{s}_{\text{H}_2}(p, T_0) - \bar{s}_{\text{H}_2}(p_0, T_0)] = -\bar{R} \ln\left(\frac{p}{p_0}\right) = -26.763 \text{ kJ/kmol K}$$

$$[\bar{s}_{\text{O}_2}(p, T_2) - \bar{s}_{\text{O}_2}(p, T_1)] = \bar{c}_p \ln\left(\frac{T_2}{T_1}\right) = 24.328 \text{ kJ/kmol K}$$

$$[\bar{s}_{\text{N}_2}(p, T_2) - \bar{s}_{\text{N}_2}(p, T_1)] = \bar{c}_p \ln\left(\frac{T_2}{T_1}\right) = 24.328 \text{ kJ/kmol K}$$

Hence,

$$\Delta\bar{S}_{\text{irrev}} = -44.27 + 27.175 - 0.5 \times 27.175 - (-26.763) + \\ 0.21 \times A \times 24.328 + 0.79 \times A \times 24.328$$

Taking $A = 7.309$,

$$\Delta\bar{S}_{\text{irrev}} = 173.9 \text{ kJ/K per kmol H}_2 \text{ supplied}$$

(d) Lost work = $T_0 \Delta\bar{S}_{\text{irrev}} = 298.15 \times 173.9 \times 10^{-3} = 51.85 \text{ MJ/kmol H}_2 \text{ supplied}$

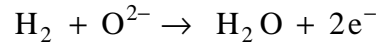
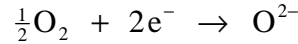
This represents a reduction in rational efficiency of,

$$\frac{T_0 \Delta\bar{S}_{\text{irrev}}}{[-\Delta\bar{G}_{298}^0]} = \frac{51.85}{228.6} = 0.227 \equiv 22.7 \%$$

The maximum rational efficiency of the GT plant is therefore 77.3 %.

A loss of 22.7 % efficiency is a high price to pay for burning fuel just to get hot gas!

7. (a) The electrochemical reactions at the cathode and anode are :



Thus, for each molecule of H_2 oxidised, 2 electrons are transferred.

From the Data Book for Reaction 5 ($\text{H}_2 + \frac{1}{2}\text{O}_2 \rightarrow \text{H}_2\text{O}$),

$$[\Delta\bar{G}_{1200}^0] = -\bar{R}T\ln(K_p) = -8314.3 \times 1200 \times 18.182 = -181.40 \times 10^6 \text{ J/kmol}$$

Faraday's constant is $F = 1.602 \times 10^{-19} \times 6.022 \times 10^{26} = 9.6472 \times 10^7 \text{ C/kmol}$.

The Gibbs potential is always based on standard pressure (1 bar).

Hence, at 1200 K and standard pressure,

$$E_G = \frac{[-\Delta\bar{G}_{1200}^0]}{2F} = \frac{181.40 \times 10^6}{2 \times 9.6472 \times 10^7} = 0.9402 \text{ volts}$$

Note that the Gibbs potential depends only on temperature. It is a reference potential (rather than the maximum output voltage possible).

- (b) At stack inlet, the mole fractions are $X_{\text{H}_2} = 0.500$, $X_{\text{H}_2\text{O}} = 0.375$, $X_{\text{CO}_2} = 0.125$. The stack operates at 5 bar, so the Nernst potential at inlet is,

$$\begin{aligned} E_N &= E_G + \frac{\bar{R}T}{2F} \left\{ \frac{1}{2} \ln\left(\frac{P}{P_0}\right) + \ln\left[\frac{(X_{\text{H}_2})(X_{\text{O}_2})^{1/2}}{(X_{\text{H}_2\text{O}})}\right] \right\} \\ &= 0.9402 + \frac{8314.3 \times 1200}{2 \times 9.6472 \times 10^7} \left\{ \frac{1}{2} \ln(5) + \ln\left[\frac{(0.500)(0.21)^{1/2}}{(0.375)}\right] \right\} \\ &= 0.9402 + 0.0416 - 0.0255 \\ &= 0.9563 \text{ volts (at stack inlet)} \end{aligned}$$

Note that $E_N > E_G$ because the stack operates at 5 bar pressure.

- (c) Every time one molecule of H_2 is oxidised, one molecule of H_2O is created. Hence the molar flowrate on the fuel side remains constant (the mass flowrate increases because of the gain of an oxygen atom). At stack outlet, 80% of the H_2 has been utilised so the mole fractions are $X_{\text{H}_2} = 0.100$, $X_{\text{H}_2\text{O}} = 0.775$, $X_{\text{CO}_2} = 0.125$. The Nernst potential at stack outlet is,

$$\begin{aligned} E_N &= 0.9402 + \frac{8314.3 \times 1200}{2 \times 9.6472 \times 10^7} \left\{ \frac{1}{2} \ln(5) + \ln\left[\frac{(0.100)(0.21)^{1/2}}{(0.775)}\right] \right\} \\ &= 0.9402 + 0.0416 - 0.1462 \\ &= 0.8356 \text{ volts (at stack outlet)} \end{aligned}$$

8. (a) Electric current through each cell (at 3000 A/m^2) is,

$$I = 3000 \times (55 \times 20) \times 10^{-6} = 3.3 \text{ amps}$$

Voltage from 20 series-connected cells operating at 85% average Nernst voltage is,

$$V = 20 \times 0.85 \times \frac{(0.9563 + 0.8356)}{2} = 15.23 \text{ volts}$$

Power delivered by module (with cells on both sides),

$$\text{Module Power} = 2 \times 3.3 \times 15.23 = 100.5 \text{ watts}$$

$$\text{Bundle voltage} = 10 \times 15.23 = 152.3 \text{ volts}$$

$$\text{Power delivered by a bundle} = 10 \times 100.5 = 1005 \text{ W} = 1.005 \text{ kW}$$

250 kW stack requires about 250 bundles.

$$\text{Approximate volume of stack} = 250 \times 10 \times [500 \times 65 \times (6 + 6)] = 975.0 \times 10^6 \text{ mm}^3$$

Stack volume is about 1 m^3

(b) H_2 consumed by each cell = $\frac{I}{2F} = \frac{3.3}{2 \times 9.6472 \times 10^7} = 1.710 \times 10^{-8} \text{ kmol/s}$

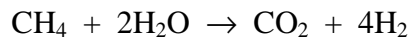
$$\text{H}_2 \text{ consumption in module} = 2 \times 20 \times 1.710 \times 10^{-8} = 6.840 \times 10^{-7} \text{ kmol/s}$$

$$\text{H}_2 \text{ consumption in stack} = 250 \times 10 \times 6.840 \times 10^{-7} = 1.710 \times 10^{-3} \text{ kmol/s}$$

- (c) Fuel utilisation is 80 %. Hence :

$$\text{Supply rate of H}_2 = \frac{1.710 \times 10^{-3}}{0.8} = 2.1375 \times 10^{-3} \text{ kmol/s}$$

If there is no CO present after reforming, then the overall steam reforming reaction is,



Hence, 1 mole of CH_4 produces 4 moles of H_2 .

$$\text{Hence, supply rate of CH}_4 = \frac{2.1375 \times 10^{-3}}{4} = 5.344 \times 10^{-4} \text{ kmol/s}$$

- (d) The rational efficiency is the actual power divided by the maximum power for the plant supplied with CH_4 (not H_2). The maximum power would be obtained in an ideal plant if the reaction $\text{CH}_4 + 2\text{O}_2 \rightarrow \text{CO}_2 + 2\text{H}_2\text{O}$ were carried out reversibly with reactants supplied, and products exhausted, separately at 25°C and 1 bar. Hence :

$$\text{Plant rational efficiency} = \frac{250 \times 10 \times 100.5}{5.344 \times 10^{-4} \times 800.0 \times 10^6} = 0.588 \cong 59 \%$$