Engineering Tripos Part IIA **THIRD YEAR**

Module 3A5 : Energy and Power Generation

GAS TURBINES AND STEAM CYCLES

Examples Paper 1

Treat air as a perfect gas with c_p = 1.01 kJ/kg K and γ = 1.40.
Take the dead state for exergy to be 25 °C and 1 bar.

Q1 In a closed circuit gas turbine plant using a perfect gas as the working fluid, the absolute temperatures at inlet to the compressor and turbine are T_1 and T_3 respectively. The plant operates with an isentropic temperature ratio of r_t and the isentropic efficiencies of the compressor and turbine are η_c and η_t respectively. Derive an expression for the ratio of the compressor work input to the turbine work output.

Evaluate this ratio when T_1 = 290 K, T_3 = 1500 K, $\eta_c = \eta_t = 0.85$, the pressure ratio is 15, and the gas is air. Neglecting mechanical losses and stray heat losses from the plant, calculate the cycle efficiency and the cycle efficiency of the corresponding air-standard Joule cycle.

Q2 A CBT open-circuit gas turbine has a compressor pressure ratio of 10.0. The air enters the compressor at a pressure of 1 bar and a temperature of 25 °C. The compressor is adiabatic and has a polytropic efficiency of 0.9. The air leaving the compressor is supplied directly to an adiabatic combustion chamber where it is mixed with methane (which enters the chamber at 25 °C) and combustion takes place at constant pressure. The products of combustion leave the chamber at 1300 °C and expand adiabatically through the turbine with a polytropic efficiency of 0.9. The turbine exhaust pressure is 1.1 bar. Calculate the air/fuel ratio by mass, the specific work output, the overall and rational efficiencies of the plant, and the exergy flowrate in the exhaust (per unit mass of air entering).

For the combustion of methane with air, ΔH_0 = –802.0 MJ/kmol and ΔG_0 = –800.0 MJ/kmol of methane. Assume that the products of combustion can be treated as a perfect gas with c_p = 1.10 kJ/kg.K and γ = 1.35.

Q3 A compressor has a fixed overall pressure ratio and a single stage of intercooling. There is negligible pressure loss in the intercooler. The LP and HP compressor have the same isentropic efficiency η_c. The LP compressor causes the air temperature to increase from T_1 to T_2 but the intercooler has an effectiveness[1] of K, which in this case means that it only cools the air from T_2 to $(T_2 - K(T_2 - T_1))$.

Show that the pressure ratio across the first compressor that minimises the total compressor work input is equal to the square root of the overall pressure ratio and is independent of η_c and K.

Q4 A closed cycle gas turbine uses a perfect gas as the working fluid. There is negligible pressure loss in the heater that, for a certain operating condition, has an outlet temperature of T_3. For this condition, the heat input is Q, the turbine work output is W_t (both per unit mass of working fluid) and the cycle efficiency is η. Maintaining the same compressor pressure ratio, the heat input is changed to $Q + \delta Q$. The heater outlet temperature is then $T_3 + \delta T_3$ and the turbine work output is $W_t + \delta W_t$.

(a) Show that the fractional change in cycle efficiency is given by,

$$\frac{\delta\eta}{\eta} = \frac{\delta W_t}{\eta Q} - \frac{\delta Q}{Q}$$

(b) Hence, assuming that the turbine polytropic efficiency does not vary with T_3, show that the rate of change of η with T_3 at constant pressure ratio r is given by,

$$\left(\frac{\partial\eta}{\partial T_3}\right)_r = \frac{1 - T_4/T_3 - \eta}{(T_3 - T_2)}$$

where T_2 is the compressor delivery temperature and T_4 is the turbine exit temperature. Calculate $(\partial\eta/\partial T_3)_r$ using the following data:

$T_1 = 300$ K, $T_3 = 1500$ K, $r = 20$, $\eta_c = \eta_t = 0.90$ (polytropic), $\gamma = 1.35$.

Q5 The gas turbine of Q2 is fitted with a heat exchanger which transfers heat from the hot exhaust gases to the air leaving the compressor. The effectiveness of the heat exchanger is 0.75. The combustor outlet temperature and the turbine exhaust temperature remain the same as Q2.

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[1] The heat exchanger effectiveness ε is actually defined as

$$\varepsilon = \frac{\text{actual heat transfer}}{\text{max possible heat transfer}} = \frac{(mc_p)_{hot}(T_{hot_{in}} - T_{hot_{out}})}{(mc_p)_{min}(T_{hot_{in}} - T_{cold_{in}})} = \frac{(mc_p)_{cold}(T_{cold_{out}} - T_{cold_{in}})}{(mc_p)_{min}(T_{hot_{in}} - T_{cold_{in}})}$$

(a) Sketch a graph of temperature vs. amount of heat transferred for the heat exchanger.

(b) Neglecting any pressure loss in the heat exchanger, calculate the temperature of the air entering the combustion chamber, the new air/fuel ratio, and the new rational efficiency of the plant.

(c) Calculate the temperature of the exhaust gas leaving the heat exchanger and the exhaust exergy flow rate (per unit mass of air entering).

Q6 A cooled gas turbine has a compressor delivery temperature T_2, a combustor outlet temperature T_3 and a turbine outlet temperature T_4. The pressure loss in the combustor and the additional mass flow rate due to the fuel may both be neglected. A small fraction δm of the compressor delivery air is bled off before the combustion chamber and is used to cool the turbine blades. This air is adiabatically throttled and is then mixed with the main turbine flow when the latter is at temperature T_m. The mixing may be assumed to take place without altering the total pressure.

(a) Assume that air and the products of combustion behave as perfect gases with the same values of c_p and γ, and also that the turbine polytropic efficiency is unaffected by the addition of the coolant. Show that the increase in turbine work output δW_t (per unit mass of air entering the compressor) is given by (the negative quantity),

$$\delta W_t = -c_p\left(T_3 - T_2 - T_4 + \frac{T_2 T_4}{T_m}\right)\delta m$$

(b) Find the reduction in heat input and then use the first equation of Q4 to show that the increase in cycle efficiency $\delta\eta$ is given by (the negative quantity),

$$\delta\eta = -\left[1 - \eta - \frac{T_4(T_m - T_2)}{T_m(T_3 - T_2)}\right]\delta m$$

(c) Combine this result with that of Q4(a) to show that the combined effect on the cycle efficiency of increasing the combustor outlet temperature and increasing the cooling flow rate is given by,

$$\left(\frac{\partial\eta}{\partial T_3}\right)_r = \frac{1 - T_4/T_3 - \eta}{(T_3 - T_2)} - \left[1 - \eta - \frac{T_4(T_m - T_2)}{T_m(T_3 - T_2)}\right]\frac{dm}{dT_3}$$

(d) Using values from Q4 and taking T_m to be $0.9T_3$, estimate the minimum value of dT_3/dm resulting in an increase in cycle efficiency.

(e) What is the other main advantage of being able to increase the turbine entry temperature?

ANSWERS

Q1 $\dfrac{r_t T_1}{\eta_c \eta_t T_3}$, 0.580, 0.355, 0.539.

Q2 45.19, 387.5 kJ/kg air, 0.350, 0.351, 345.0 kJ/kg air.

Q3 –

Q4 $(\partial\eta/\partial T_3)_r = 8.6 \times 10^{-5}\ K^{-1}$ (*i.e.* an increase in η of 0.0086 for an increase in T_3 of 100 K)

Q5 859.8 K, 58.33, 0.449, 722.9 K, 188.0 kJ/kg air.

Q6 dT_3/dm = 1367.0 K (*i.e.*, an increase in T_3 of 13.67 K for a 1% increase in cooling air)

PAST TRIPOS QUESTIONS FOR REVISION

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