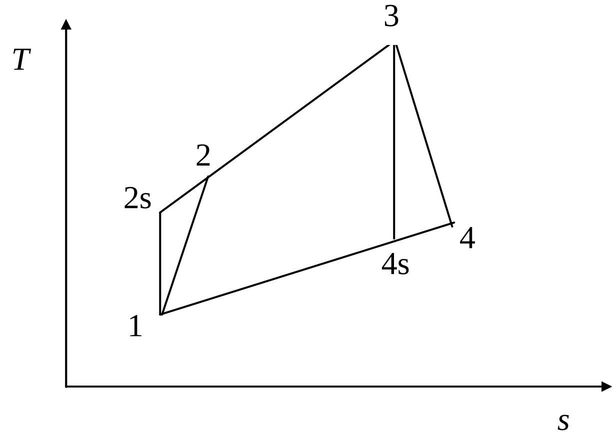
Engineering Tripos Part IIA **THIRD YEAR**

Module 3A5: Energy and Power Generation

GAS TURBINE AND STEAM CYCLES

Solutions to Examples Paper 1 by HP Hodson, October 2004

1.



Compressor work, $w_c = \dfrac{c_p}{\eta_c}(T_{2s} - T_1) = \dfrac{c_p T_1}{\eta_c}(r_t - 1)$

Turbine work, $w_t = \eta_t \, c_p \, (T_3 - T_{4s}) = \eta_t \, c_p \, T_3 \left(1 - \dfrac{1}{r_t}\right)$

Therefore, $\dfrac{w_c}{w_t} = \dfrac{r_t \, T_1}{\eta_c \, \eta_t \, T_3}$

For $r_p = 15$, $r_t = (r_p)^{(\gamma-1)/\gamma} = 2.168$, $T_{2s} = 628.72$ K, $T_2 = 688.49$ K.

$\dfrac{w_c}{w_t} = \dfrac{2.168 \times 290.0}{0.85 \times 0.85 \times 1500.0} = 0.580$ (More than 50% turbine work taken by compressor)

$$w_c = \frac{c_p T_1}{\eta_c}(r_t - 1) = \frac{1.01 \times 290.0 \times 1.168}{0.85} = 402.48 \text{ kJ/kg}$$

$$w_t = \eta_t \, c_p \, T_3 \left(1 - \frac{1}{r_t}\right) = 0.85 \times 1.01 \times 1500.0 \times \left(1 - \frac{1}{2.168}\right) = 693.77 \text{ kJ/kg}$$

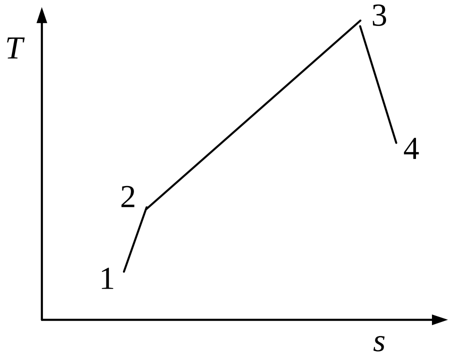
$$q_{in} = c_p (T_3 - T_2) = 1.01 \times (1500.0 - 688.49) = 819.63 \text{ kJ/kg}$$

Cycle efficiency, $\eta_{cycle} = \dfrac{693.77 - 402.48}{819.63} = 0.355$

Joule cycle efficiency (same pressure ratio), $\eta_{Joule} = 1 - \dfrac{1}{r_t} = 0.539$

(Effects of non-isentropic turbomachinery are very significant)

2.



For T_1 = 298.15 K, pressure ratio = 10, polytropic efficiency = 0.9, γ = 1.40,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma\,\eta_c}} = 10^{0.3175} = 2.077 \qquad \rightarrow \quad T_2 = 619.3\ \text{K}$$

For T_3 = 1573.15 K, pressure ratio = 10/1.1 = 9.091, polytropic efficiency = 0.9, γ = 1.35,

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{(\gamma-1)\eta_t}{\gamma}} = 9.091^{0.2333} = 1.674 \qquad \rightarrow \quad T_4 = 940.0\ \text{K}$$

SFEE for the combustor (A = air/fuel ratio), $-\Delta H_0 = 802\times10^3 / 16 = 50.125\times10^3$ kJ/kg,

$$(\dot{m}_a + \dot{m}_f)h_{p3} - \dot{m}_a h_{a2} - \dot{m}_f h_{f0} = 0$$
$$(\dot{m}_a + \dot{m}_f)(h_{p3} - h_{p0}) - \dot{m}_a(h_{a2} - h_{a0}) + (\dot{m}_a + \dot{m}_f)h_{p0} - \dot{m}_a h_{a0} - \dot{m}_f h_{f0} = 0$$
$$(A+1)c_{pp}(T_3 - T_0) - Ac_{pa}(T_2 - T_0) + \Delta H_0 = 0$$

$$A = \frac{-\Delta H_0 - c_{pp}(T_3 - T_0)}{c_{pp}(T_3 - T_0) - c_{pa}(T_2 - T_0)} = \frac{50.125\times10^3 - 1.10\times(1573.15 - 298.15)}{1.10\times(1573.15 - 298.15) - 1.01\times(619.3 - 298.15)} = 45.19$$

(Stoichiometric air/fuel ratio is 17.2 so 163% excess air is required to give T_3 = 1300 °C)

Working per unit mass of air entering, (f = fuel/air ratio = 1/A = 0.0221),

Compressor work = $w_c = c_{pa}(T_2 - T_1) = 1.01\times(619.3 - 298.15) = 324.4$ kJ/kg

Turbine work = $w_t = (1+f)c_{pp}(T_3 - T_4) = 1.0221\times1.10\times(1573.15 - 940.0) = 711.9$ kJ/kg

Specific work output = $w_x = w_t - w_c = 387.5$ kJ/kg air.

Overall efficiency = $\eta_{ov} = \dfrac{w_x}{f(-\Delta H_0)} = \dfrac{387.5}{0.0221\times50.125\times10^3} = 0.350$

Rational efficiency = $\eta_{rat} = \dfrac{w_x}{f(-\Delta G_0)} = \dfrac{387.5}{0.0221\times50.0\times10^3} = 0.351$

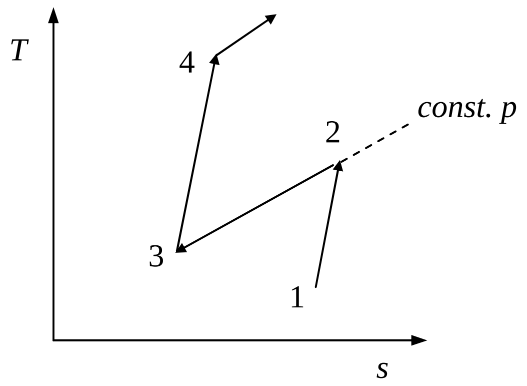
Exhaust exergy flowrate = $(1+f)[(h_{p4} - T_0 s_{p4}) - (h_{p0} - T_0 s_{p0})]$

$$= (1+f)\left\{c_{pp}(T_4 - T_0) - c_{pp}T_0\,\ell n\left(\frac{T_4}{T_0}\right) + R_p T_0\,\ell n\left(\frac{p_4}{p_0}\right)\right\}$$

= 1.0221 × (706.04 – 376.60 + 8.10)

= 345.0 kJ/kg air (similar in magnitude to w_x)

3.



Overall isentropic temperature ratio $= \; r = \left(\dfrac{p_4}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$

Compressor 1 isentropic temperature ratio $= \; r_1 = \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \;\rightarrow\; T_2 = T_1 + \dfrac{T_1(r_1-1)}{\eta_c}$

Intercooler outlet temperature $= \; T_3 = T_2 - K(T_2 - T_1) = T_1\left[1 + \dfrac{(1-K)(r_1-1)}{\eta_c}\right]$

Compressor 1 work $= \; w_{c1} = \dfrac{c_p T_1(r_1-1)}{\eta_c}$

Compressor 2 work $= \; w_{c2} = \dfrac{c_p T_3(r_2-1)}{\eta_c} = \dfrac{c_p T_1(r_2-1)}{\eta_c}\left[1 + \dfrac{(1-K)(r_1-1)}{\eta_c}\right]$

Total work $= \; w_c = \dfrac{c_p T_1}{\eta_c}\left[(r_1-1) + (r_2-1) + \dfrac{(1-K)(r_1-1)(r_2-1)}{\eta_c}\right]$

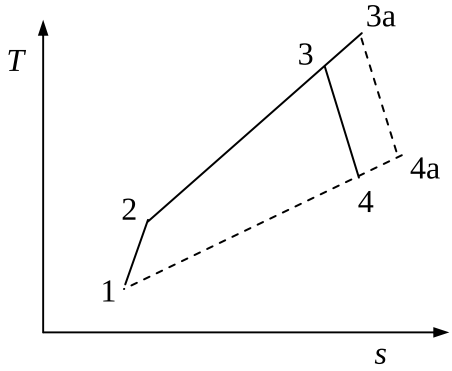
$$\therefore \quad \frac{dw_c}{dr_1} = \frac{c_p T_1}{\eta_c}\left[1 + \frac{dr_2}{dr_1} + \frac{(1-K)(r_2-1)}{\eta_c} + \frac{(1-K)(r_1-1)}{\eta_c}\frac{dr_2}{dr_1}\right]$$

Now $r_1 r_2 = r \;\rightarrow\; \dfrac{dr_2}{dr_1} = -\dfrac{r_2}{r_1}$. Hence,

$$\frac{dw_c}{dr_1} = \left(\frac{r_1 - r_2}{r_1}\right)\frac{c_p T_1}{\eta_c}\left[1 - \frac{(1-K)}{\eta_c}\right]$$

For minimum compressor work, $r_1 = r_2 = \sqrt{r}$ (independent of K and η_c)

4.



Cycle efficiency $= \eta = \dfrac{W_t - W_c}{Q}$ $\quad\rightarrow\quad$ $\eta Q = (W_t - W_c)$

Take logs and differentiate keeping w_c constant $\quad\rightarrow\quad$ $\dfrac{\delta\eta}{\eta} = \dfrac{\delta W_t}{\eta Q} - \dfrac{\delta Q}{Q}$

Now, $Q = c_p(T_3 - T_2)$. Thus, $\delta Q = c_p\,\delta T_3$ as T_2 is constant.

Also, $W_t = c_p(T_3 - T_4)$. Thus, $\delta W_t = c_p\left[1 - \left(\dfrac{\partial T_4}{\partial T_3}\right)_r\right]\delta T_3$

Assuming η_t (the turbine polytropic efficiency) does not change with T_3,

$$\frac{T_4}{T_3} = \left(\frac{1}{r}\right)^{\frac{(\gamma-1)\eta_t}{\gamma}} \quad\rightarrow\quad \left(\frac{\partial T_4}{\partial T_3}\right)_r = \left(\frac{1}{r}\right)^{\frac{(\gamma-1)\eta_t}{\gamma}} = \frac{T_4}{T_3}$$

Hence,

$$\frac{\delta\eta}{\eta} = \frac{c_p\,(1 - T_4/T_3)\delta T_3}{\eta\, c_p\,(T_3 - T_2)} - \frac{c_p\,\delta T_3}{c_p\,(T_3 - T_2)}$$

In the limit as $\delta Q \rightarrow 0$,

$$\left(\frac{\partial\eta}{\partial T_3}\right)_r = \frac{1 - T_4/T_3 - \eta}{(T_3 - T_2)}$$

For $r = 20$, $T_1 = 300$ K, $\eta_c = 0.90$, $\gamma = 1.35$ $\quad\rightarrow\quad$ $T_2 = T_1\, r^{(\gamma-1)/\gamma\eta_c} = 711.1$ K

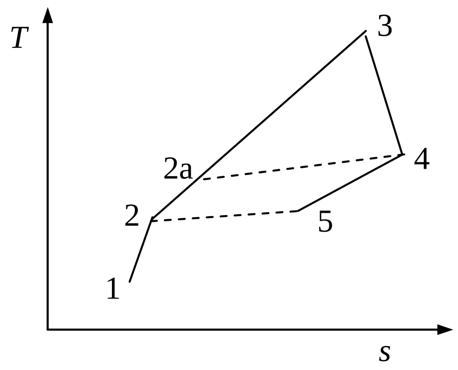
For $r = 20$, $T_3 = 1500$ K, $\eta_t = 0.90$, $\gamma = 1.35$ $\quad\rightarrow\quad$ $T_4 = T_3\, r^{-(\gamma-1)\eta_t/\gamma} = 745.6$ K

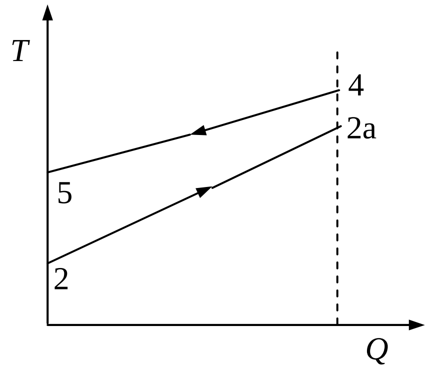
Cycle efficiency $= \eta = \dfrac{(T_3 - T_4) - (T_2 - T_1)}{(T_3 - T_2)} = \dfrac{(1500.0 - 745.6) - (711.1 - 300.0)}{(1500.0 - 711.1)} = 0.435$

$$\therefore \qquad \left(\frac{\partial\eta}{\partial T_3}\right)_r = \frac{1 - 745.6/1500.0 - 0.435}{(1500.0 - 711.1)} = 0.000086$$

This implies an increase in cycle efficiency of 0.86 percentage points for an increase in T_3 of 100 K (which is not a good return for what would be a very costly research effort). In fact, the potential gain in efficiency decreases dramatically as T_3 gets high and, if the effects of varying c_p are included, $(\partial\eta/\partial T_3)_r$ actually becomes negative at very high T_3.

5.





$\dot{m}c_p$ is greater for the exhaust gas and hence the slope of the line on the *T*-*Q* diagram is less.

Heat exchanger effectiveness is given by,

$$\varepsilon = \frac{T_{2a} - T_2}{T_4 - T_2}$$

$$\therefore \quad T_{2a} = T_2 + \varepsilon(T_4 - T_2) = 619.3 + 0.75 \times (940.0 - 619.3) = 859.8 \text{ K}$$

New air/fuel ratio,

$$A = \frac{-\Delta H_0 - c_{pp}(T_3 - T_0)}{c_{pp}(T_3 - T_0) - c_{pa}(T_{2a} - T_0)} = \frac{50.125\times10^3 - 1.10\times(1573.15 - 298.15)}{1.10\times(1573.15-298.15) - 1.01\times(859.8 - 298.15)} = 58.33$$

Working per unit mass of air entering, (f = fuel/air ratio = 1/A = 0.0171),

Compressor work = $w_c = c_{pa}(T_2 - T_1) = 1.01\times(619.3 - 298.15) = 324.4$ kJ/kg

Turbine work = $w_t = (1+f)c_{pp}(T_3 - T_4) = 1.0171\times1.10\times(1573.15 - 940.0) = 708.37$ kJ/kg

Specific work output = $w_x = w_t - w_c = 384.0$ kJ/kg air.

Rational efficiency = $\eta_{rat} = \dfrac{w_x}{f(-\Delta G_0)} = \dfrac{384.0}{0.0171\times50.0\times10^3} = 0.449$

(Large increase in rational efficiency by fitting a heat exchanger)

SFEE for the heat exchanger: $c_{pa}(T_{2a} - T_2) = (1+f)c_{pp}(T_4 - T_5)$

$$\therefore \quad T_5 = T_4 - \frac{c_{pa}(T_{2a} - T_2)}{(1+f)c_{pp}} = 940.0 - \frac{1.01\times(859.8 - 619.3)}{1.0171\times1.10} = 722.9 \text{ K}$$

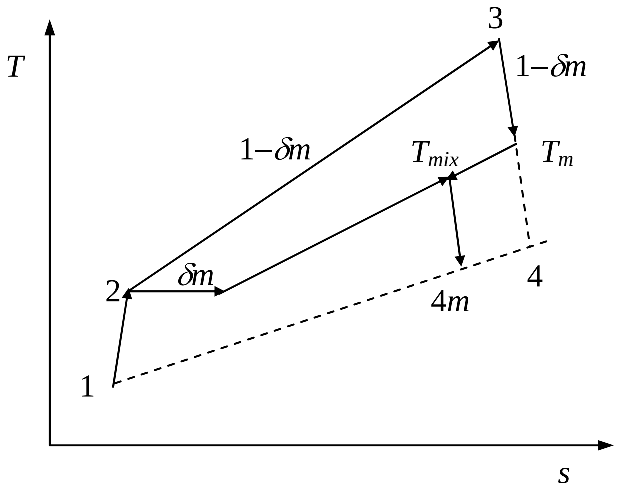
Exhaust exergy flowrate = $(1+f)[(h_{p5} - T_0 s_{p5}) - (h_{p0} - T_0 s_{p0})]$

$$= (1+f)\left\{c_{pp}(T_5 - T_0) - c_{pp}T_0\,\ell n\left(\frac{T_5}{T_0}\right) + R_p T_0\,\ell n\left(\frac{p_5}{p_0}\right)\right\}$$

$= 1.0171 \times (467.22 - 290.47 + 8.10)$

$= 188.0$ kJ/kg air (very much reduced)

6.



Increase of work due to δm not participating in first expansion $= \delta W_{t1} = -c_p(T_3 - T_m)\delta m$

Increase of work in second expansion $= \delta W_{t2} = c_p(T_{mix} - T_{4m}) - c_p(T_m - T_4)$

If the turbine polytropic efficiency is unchanged then, because the pressure ratio is the same,

$$\frac{T_{mix}}{T_{4m}} = \frac{T_m}{T_4}$$

$$\therefore \qquad \delta W_{t2} = c_p T_{mix}\left(1 - \frac{T_{4m}}{T_{mix}}\right) - c_p T_m \left(1 - \frac{T_4}{T_m}\right) = -c_p(T_m - T_{mix})\left(1 - \frac{T_4}{T_m}\right)$$

SFEE for the mixing process,

$$(1-\delta m)c_p T_m + \delta m\, c_p T_2 = c_p T_{mix} \quad \rightarrow \quad (T_m - T_{mix}) = \delta m (T_m - T_2)$$

$$\therefore \qquad \delta W_{t2} = -c_p(T_m - T_2)\left(1 - \frac{T_4}{T_m}\right)\delta m$$

Total increase in turbine work is therefore,

$$\delta W_t = \delta W_{t1} + \delta W_{t2} = -c_p\left(T_3 - T_2 - T_4 + \frac{T_2 T_4}{T_m}\right)\delta m$$

Increase in heat input $= \delta Q = -c_p(T_3 - T_2)\delta m$

It has been shown previously that,

$$\frac{\delta\eta}{\eta} = \frac{\delta W_t}{\eta Q} - \frac{\delta Q}{Q} \quad \rightarrow \quad \delta\eta = \frac{\delta W_t - \eta\delta Q}{Q}$$

Substituting the expressions for δW_t and δQ gives,

$$\delta\eta = -\left[\frac{T_3 - T_2 - T_4 + T_2 T_4 / T_m - \eta(T_3 - T_2)}{(T_3 - T_2)}\right]\delta m = -\left[1 - \eta - \frac{T_4(T_m - T_2)}{T_m(T_3 - T_2)}\right]\delta m$$

For changes both in T_3 and cooling flowrate (at constant pressure ratio),

$$\delta\eta = \left(\frac{\partial\eta}{\partial T_3}\right)_m \delta T_3 + \left(\frac{\partial\eta}{\partial m}\right)_{T_3} \delta m$$

$$\therefore \qquad \left(\frac{\partial\eta}{\partial T_3}\right)_r = \left(\frac{\partial\eta}{\partial T_3}\right)_m + \left(\frac{\partial\eta}{\partial m}\right)_{T_3} \frac{dm}{dT_3}$$

$$\left(\frac{\partial\eta}{\partial T_3}\right)_r = \left[\frac{1-\eta - T_4/T_3}{(T_3 - T_2)}\right] - \left[1 - \eta - \frac{T_4(T_m - T_2)}{T_m(T_3 - T_2)}\right]\frac{dm}{dT_3}$$

The first term on the RHS represents an increase in efficiency due to increasing the COT. The second term represents a reduction in efficiency due to the increased cooling flowrate required. All changes are at constant pressure ratio.

We have, $T_2 = 711.1$ K, $T_3 = 1500$ K, $T_4 = 745.6$ K, $\eta = 0.435$, $T_m = 0.9T_3 = 1350.0$ K.

$$\therefore \qquad \left(\frac{\partial\eta}{\partial T_3}\right)_r = 0.0000861 - 0.1177\frac{dm}{dT_3}$$

For the efficiency to increase with increase of T_3 and the extra cooling,

$$\left(\frac{\partial\eta}{\partial T_3}\right)_r = 0.0000861 - 0.1177\frac{dm}{dT_3} > 0 \qquad \rightarrow \qquad \frac{dT_3}{dm} > \frac{0.1177}{0.0000861} = 1367.0$$

Hence, for every 1% of cooling air used, it is necessary to be able to increase the combustor outlet temperature by more than 13.67 K if the cycle efficiency is not to decrease.

The other advantage of increasing the COT is the increase in specific work output.