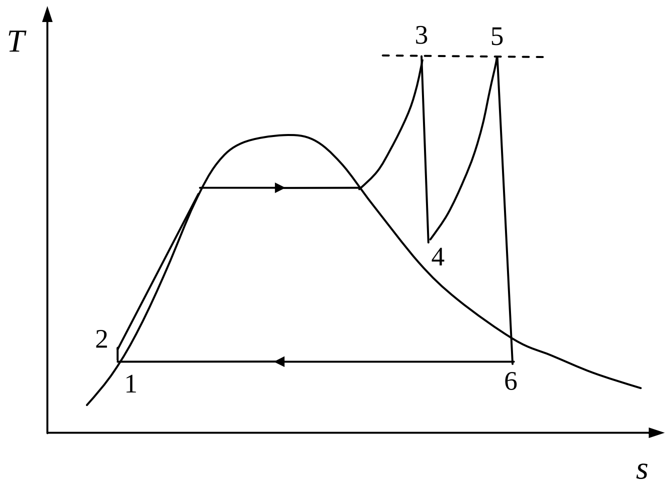
**Engineering Tripos Part IIA** **THIRD YEAR**

**Module 3A5 : Energy and Power Generation**

**GAS TURBINE AND STEAM CYCLES**

**Solutions to Examples Paper 2 by HP Hodson, October 2004**

**1**.




From the tables, $h_3 = 3448.0$ kJ/kg, $s_3 = 6.521$ kJ/kgK

From the chart, $h_{4s} = 3055$ kJ/kg (3054.8 from interpolation in the tables)

HP Turbine work $= \eta_{hp}(h_3 - h_{4s}) = 0.9\times(3448.0 - 3054.8) = 353.9$ kJ/kg

$\therefore \quad h_4 = 3448.0 - 353.9 = 3094.1$ kJ/kg

From the tables, $h_5 = 3559.0$ kJ/kg, $s_5 = 7.233$ kJ/kgK

From the chart, $h_{6s} = 2208$ kJ/kg (2205.7 from the tables)

IP/LP Turbine work $= \eta_{lp}(h_5 - h_{6s}) = 0.85\times(3559.0 - 2205.7) = 1150.3$ kJ/kg

$\therefore \quad h_6 = 3559.0 - 1150.3 = 2408.7$ kJ/kg

From the tables, $h_1 = 137.8$ kJ/kg, $\rho_{liq} = 1/0.001005 = 995.0$ kg/m$^3$

Feed Pump work $\cong \dfrac{p_2 - p_1}{\eta_{fp}\,\rho_{liq}} = \dfrac{(150 - 0.05)\times10^5}{0.7\times995.0} = 2.15\times10^4$ J/kg $= 21.5$ kJ/kg

$\therefore \quad h_2 = 137.8 + 21.5 = 159.3$ kJ/kg

Specific work output $= W_x = 353.9 + 1150.3 - 21.5 = 1482.7$ kJ/kg

Without reheat, from worked example in lectures, $W_x = 1219.7$ kJ/kg

Hence reheat increases the specific work by 21%.

Power output is 500 MW, so mass flowrate circulating through the boiler is,

$$\dot{m} = \frac{500 \times 10^3}{1482.7} = 337.2 \text{ kg/s} \qquad \text{(which is a lot of steam!)}$$

HP turbine power = $337.2 \times 353.9$ = $119.3 \times 10^3$ kW = 119.3 MW

IP/LP turbine power = $337.2 \times 1150.3$ = $387.9 \times 10^3$ kW = 387.9 MW

Feed pump power = $337.2 \times 21.5$ = $7.2 \times 10^3$ kW = 7.2 MW

(The feed pump power, although small compared to the turbine power, is still substantial.)

$Q_{in}$ main boiler = $h_3 - h_2$ = $3448.0 - 159.3$ = 3288.7 kJ/kg

$Q_{in}$ reheater = $h_5 - h_4$ = $3559.0 - 3094.1$ = 464.9 kJ/kg

$Q_{out}$ condenser = $h_6 - h_1$ = $2408.7 - 137.8$ = 2270.9 kJ/kg

$\dot{Q}_{in}$ main boiler = $337.2 \times 3288.7$ = $1108.9 \times 10^3$ kW = 1108.9 MW

$\dot{Q}_{in}$ reheater = $337.2 \times 464.9$ = $156.8 \times 10^3$ kW = 156.8 MW

$\dot{Q}_{out}$ condenser = $337.2 \times 2270.9$ = $765.7 \times 10^3$ kW = 765.7 MW

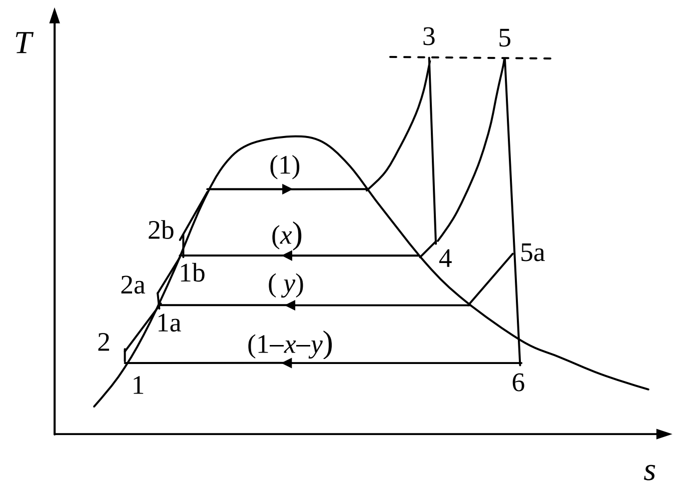
(Note the huge heat transfer rates.)

$$\text{Cycle efficiency} = \frac{W_x}{Q_{in}} = \frac{353.9 + 1150.3 - 21.5}{3288.7 + 464.9} = 0.395$$

Without reheat, from worked example in lectures, $\eta_c = 0.371$

Hence reheat increases the cycle efficiency by 2.4 percentage points.

**2.**




Condenser, $p_1 = 0.05$ bar $\rightarrow$ From question 1, $h_1 = 137.8$ kJ/kg, $\rho_{1a} = 995.0$ kg/m$^3$

Feedheater A, $p_{1a} = 4$ bar $\rightarrow$ From the tables, $h_{1a} = 604.7$ kJ/kg, $\rho_{1a} = 922.5$ kg/m$^3$

Feedheater B, $p_{1b} = 40$ bar $\rightarrow$ From the tables, $h_{1b} = 1087.4$ kJ/kg, $\rho_{1a} = 798.7$ kg/m$^3$

Note that the approximately equal enthalpy (and temperature) rises across the feedheaters should give close to maximum cycle efficiency (for two feedheaters).

For the feed pumps :

$$h_2 - h_1 = \frac{(4-0.05)\times 10^5}{0.7\times 995.0} = 0.6\times 10^3 \text{ J/kg} = 0.6 \text{ kJ/kg} \rightarrow h_2 = 138.4 \text{ kJ/kg}$$

$$h_{2a} - h_{1a} = \frac{(40-4)\times 10^5}{0.7\times 922.5} = 5.6\times 10^3 \text{ J/kg} = 5.6 \text{ kJ/kg} \rightarrow h_{2a} = 610.3 \text{ kJ/kg}$$

$$h_{2b} - h_{1b} = \frac{(150-40)\times 10^5}{0.7\times 798.7} = 19.7\times 10^3 \text{ J/kg} = 19.7 \text{ kJ/kg} \rightarrow h_{2b} = 1107.1 \text{ kJ/kg}$$

From question 1, $h_4 = 3094.1$ kJ/kg

Assuming a straight line expansion on the ($h$-$s$) diagram, $h_{5a} = 3048$ kJ/kg

SFEE for feedheater A :

$$xh_4 + (1-x)h_{2a} = h_{1b} \rightarrow x = \frac{h_{1b} - h_{2a}}{h_4 - h_{2a}} = \frac{1087.4 - 610.3}{3094.1 - 610.3} = 0.1921$$

SFEE for feedheater B :

$$yh_{5a} + (1-x-y)h_2 = (1-x)h_{1a} \rightarrow y = \frac{(1-x)(h_{1a} - h_2)}{h_{5a} - h_2} = \frac{0.8079\times(604.7 - 138.4)}{(3048.0 - 138.4)} = 0.1295$$

Per kg circulating through the boiler :

HP Turbine work = 353.9 kJ/kg (same as question 1)

IP/LP Turbine work = $(1-x)(h_5 - h_{5a}) + (1-x-y)(h_{5a} - h_6)$

= 0.8079×(3559.0 – 3048.0) + 0.6784×(3048.0 – 2408.7)

= 846.5 kJ/kg

Feed pump work = $(h_{2b} - h_{1b}) + (1-x)(h_{2a} - h_{1a}) + (1-x-y)(h_2 - h_1)$

= 19.7 + 0.8079×5.6 + 0.6784×0.6 = 24.6 kJ/kg

Specific work output = 353.9 + 846.5 – 24.6 = 1175.8 kJ/kg

Compare this with : 1219.7 kJ/kg (no feedheating, no reheat)

1482.7 kJ/kg (no feedheating, single reheat)

Feedheating has reduced the specific work by 21% from question 1.

Power output is 500 MW, so mass flowrate circulating through the boiler is,

$$\dot{m} = \frac{500\times10^3}{1175.8} = 425.2 \text{ kg/s} \qquad \text{(26\% more steam required than question 1)}$$

HP turbine power = 425.2 × 353.9 = 150.5 × $10^3$ kW = 150.5 MW

IP/LP turbine power = 425.2 × 846.5 = 359.9 × $10^3$ kW = 359.9 MW

Feed pump power = 425.2 × 24.6 = 10.4 × $10^3$ kW = 10.4 MW

$Q_{in}$ main boiler = $(h_3 - h_{2b})$ = 3448.0 – 1107.1 = 2340.9 kJ/kg

$Q_{in}$ reheater = $(1 - x)(h_5 - h_4)$ = 0.8079 × (3559.0 – 3094.1) = 375.6 kJ/kg

$Q_{out}$ condenser = $(1 - x - y)(h_6 - h_1)$ = 0.6784 × (2408.7 – 137.8) = 1540.6 kJ/kg

$\dot{Q}_{in}$ main boiler = 425.2 × 2340.9 = 995.4 × $10^3$ kW = 995.4 MW

$\dot{Q}_{in}$ reheater = 425.2 × 375.6 = 159.7 × $10^3$ kW = 159.7 MW

$\dot{Q}_{out}$ condenser = 425.2 × 1540.6 = 655.1 × $10^3$ kW = 655.1 MW

The main change from question 1 is that a substantially smaller condenser is required.

$$\text{Cycle efficiency} = \frac{W_x}{Q_{in}} = \frac{353.9 + 846.5 - 24.6}{2340.9 + 375.6} = 0.433$$

From question 1 (single reheat, no feedheating), $\eta_c$ = 0.395

Hence 2 feedheaters increase the cycle efficiency by 3.8 percentage points.

**3.** **(a)** Mass fraction of $O_2$ in air $= \dfrac{0.21\times 32}{0.21\times 32 + 0.79\times 28} = 0.233$

Mass fraction of $N_2$ in air $= \dfrac{0.79\times 28}{0.21\times 32 + 0.79\times 28} = 0.767$

Stoichiometric air requirement per kg oil :

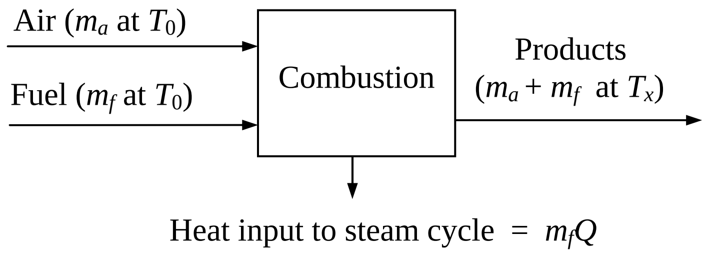
0.85 kg carbon require $\dfrac{0.85}{12}\times 32 = 2.2667$ kg of $O_2$

0.15 kg hydrogen require $\dfrac{0.15}{2}\times 16 = 1.20$ kg of $O_2$

1 kg oil requires $\dfrac{2.2667+1.20}{0.233} = 14.878$ kg air for stoichiometric combustion.

With 10% excess air, actual air/fuel ratio = 14.878 × 1.1 = 16.366

**(b)** No preheater :




Write actual SFEE and SFEE if products are discharged at $T_0$ = 25 °C,

$$m_f Q = m_a h_{a0} + m_f h_{f0} - (m_a + m_f) h_{px}$$

$$m_f(-\Delta H_0) = m_a h_{a0} + m_f h_{f0} - (m_a + m_f) h_{p0}$$

Divide each equation by $m_f(-\Delta H_0)$ and subtract to give the boiler efficiency,

$$\eta_{boiler} = \frac{Q}{(-\Delta H_0)} = 1 - \frac{(A+1)(h_{px} - h_{p0})}{(-\Delta H_0)} = 1 - \frac{(A+1)c_{pp}(T_x - T_0)}{(-\Delta H_0)}$$

where $A = m_a/m_f$ is the air/fuel ratio.

Feed water is supplied to the boiler as saturated liquid at 40 bar. From tables, $T_{fw}$ = 250.3 °C. Hence, without a preheater, the stack temperature is $T_x$ = 250.3 + 30.0 = 280.3 °C = 553.5 K. Taking $c_{pp}$ = 1.20 kJ/kgK,

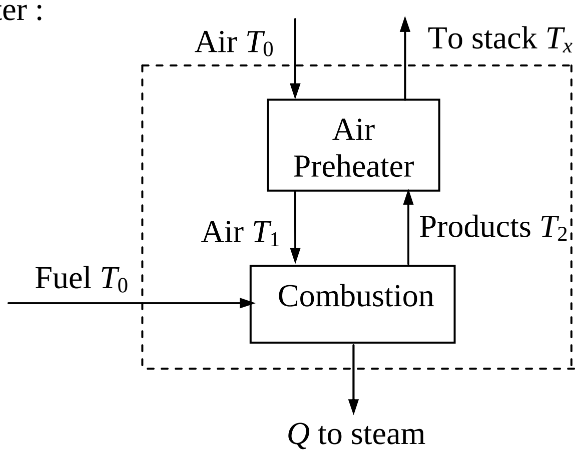
$$\eta_{boiler} = 1 - \frac{17.366\times 1.20\times(553.5-298.15)}{40.0\times 10^3} = 0.870$$

Steam plant efficiency,

$$\eta_{plant} = \eta_{boiler}\times\eta_{cycle} = 0.870\times 0.433 = 0.377$$

The poor boiler efficiency reduces the plant efficiency by 5.6 percentage points below its theoretical maximum value.

**(c)** With preheater :




Products leave boiler at $T_2$ = 250.3 + 30.0 = 280.3 °C = 553.5 K

Preheater effectiveness $= 0.7 = \dfrac{(T_1 - T_0)}{(T_2 - T_0)}$ (N.B., Based on stream with lower $mc_p$)

Hence, $T_1 = 298.15 + 0.7 \times (553.5 - 298.15) = 476.9\text{ K}$ (203.7 °C)

SFEE for preheater : $m_a c_{pa}(T_1 - T_0) = (m_a + m_f) c_{pp}(T_2 - T_x)$

Stack temperature, $T_x = T_2 - \dfrac{A c_{pa}(T_1 - T_0)}{(A+1) c_{pp}}$

$$= 553.5 - \frac{16.366 \times 1.01 \times (476.9 - 298.15)}{17.366 \times 1.20} = 411.7\text{ K} = 138.6\text{ °C}$$

(This is about the lowest acceptable value if the fuel contains sulphur.)

Writing the SFEE for the dotted control volume gives an equation identical to that in part (a). Hence,

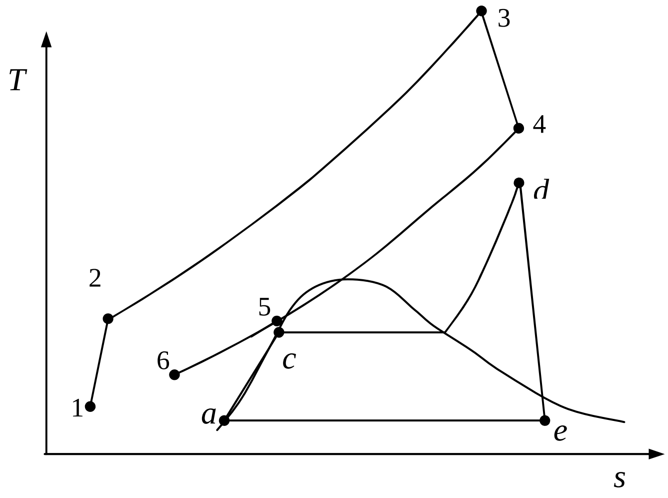
$$\eta_{boiler} = 1 - \frac{17.366 \times 1.20 \times (411.7 - 298.15)}{40.0 \times 10^3} = 0.941$$

Steam plant efficiency,

$$\eta_{plant} = \eta_{boiler} \times \eta_{cycle} = 0.941 \times 0.433 = 0.407$$

Installing an air preheater raises the plant efficiency by 3.0 percentage points.

**4.**




$T_4 = 530.0\ °C = 803.1\ K$ $\quad$ $T_d = 450.0\ °C = 723.1\ K$

$T_c = T_{sat}(40\ bar) = 250.3\ °C = 523.4\ K$ $\quad$ $T_5 = T_c + \Delta T_p = 523.4 + 10.0 = 533.4\ K$

From the steam tables, $h_d = 3331$ kJ/kg, $h_c = 1087.4$ kJ/kg, $h_a = 125.7$ kJ/kg.

If the gas turbine exhaust behaves as a perfect gas,

$$\frac{T_4 - T_5}{T_4 - T_6} = \frac{h_d - h_c}{h_d - h_a} = \frac{3331 - 1087.4}{3331 - 125.7} = 0.700$$

Stack temperature, $T_6 = T_4 - \dfrac{T_4 - T_5}{0.700} = 803.1 - \dfrac{803.1 - 533.4}{0.700} = 417.8\,\text{K} = 144.7\ °C$

HRSG (boiler) efficiency $= \eta_b = \dfrac{T_4 - T_6}{T_4 - T_0} = \dfrac{803.1 - 417.8}{803.1 - 298.1} = 0.763$

Definition of GT overall efficiency, $\eta_1 = \dfrac{P_1}{m_f(-\Delta H_0)} = 0.36$ $\quad$ ($P_1 = 100$ MW)

Hence, $m_f(-\Delta H_0) = \dfrac{100}{0.36} = 277.8\,\text{MW}$

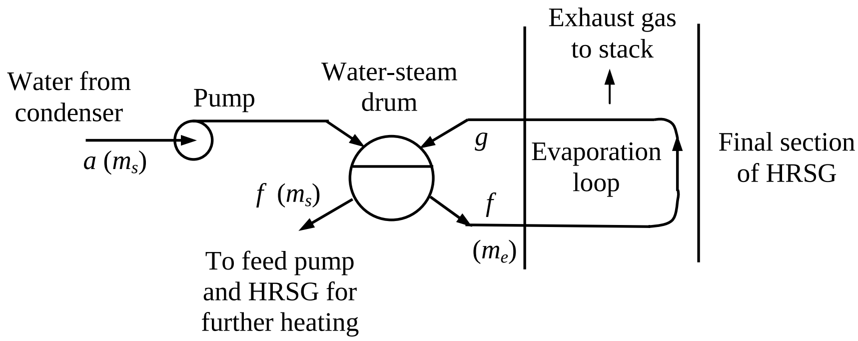
Heat transferred to steam cycle, $Q_2 = \eta_b(1 - \eta_1)[m_f(-\Delta H_0)]$

$= 0.763 \times 0.64 \times 277.8 = 135.7\,\text{MW}$

Power output from steam cycle, $P_2 = \eta_2 Q_2 = 0.32 \times 135.7 = 43.4$ MW

Combined-cycle efficiency, $\eta_{cc} = \dfrac{P_1 + P_2}{m_f[-\Delta H_0]} = \dfrac{100.0 + 43.4}{277.8} = 0.516$

Check: $\eta_{cc} = \eta_1 + \eta_b \eta_2 (1 - \eta_1) = 0.36 + 0.763 \times 0.32 \times 0.64 = 0.516$




The water-steam drum operates at a pressure of 1 atm ($T_{sat}$ = 100.0 °C).

From the tables, $h_f$ = 419.1 kJ/kg, $h_g$ = 2676.0 kJ/kg.

SFEE for the drum (neglecting the feed pump work),

$$m_s h_a + m_e h_g = m_s h_f + m_e h_f$$

$$\frac{m_e}{m_s} = \frac{h_f - h_a}{h_g - h_f} = \frac{419.1 - 125.7}{2676.0 - 419.1} = 0.130$$

There is no change in stack temperature as exactly the same heat is extracted from the exhaust gas in the HRSG. The overall efficiency also remains the same.

When extra steam is generated by extracting more heat in the HRSG, the stack temperature falls to $T_6'$ = 100.0 + 10.0 = 110.0 °C. Previously, the heat extracted was $Q_2$ = 135.7 MW.

$$\text{Extra heat extracted} = \left(\frac{T_6 - T_6'}{T_4 - T_6}\right) Q_2 = \left(\frac{144.7 - 110.0}{530.0 - 144.7}\right) \times 135.7 = 12.22 \text{ MW}$$

Conversion efficiency via the steam turbine is only 15% because the heat is supplied at low temperature. (The 15% was obtained by assuming the steam expanded from 1 atm to condenser pressure with an isentropic efficiency of about 0.85.)

Extra power output from steam turbine = 0.15 × 12.22 = 1.83 MW

$$\text{New combined-cycle overall efficiency} = \frac{100.0 + 43.4 + 1.8}{277.8} = 0.523$$

Not a dramatic increase but the extra power is almost free once the drum has been installed.

**5.** Specific work output is given by,

$$w_x = w_t - w_c = c_p(T_3 - T_4) - c_p(T_2 - T_1) = c_p\,\eta_t\,(T_3 - T_{4s}) - \frac{c_p\,(T_{2s} - T_1)}{\eta_c}$$

$$\therefore \qquad \frac{w_x}{c_p T_1} = \left[\eta_t\,\theta\left(1 - \frac{1}{\tau}\right) - \frac{1}{\eta_c}(\tau - 1)\right]$$

Differentiating with respect to $\tau$ at constant $\theta$, $\eta_c$ and $\eta_t$ gives,

$$\frac{\partial}{\partial\tau}\left(\frac{w_x}{c_p T_1}\right) = \frac{\eta_t\,\theta}{\tau^2} - \frac{1}{\eta_c}$$

Hence, maximum specific work corresponds to an isentropic temperature ratio given by,

$$\tau_w^2 = \eta_c\,\eta_t\,\theta$$

Cycle efficiency is given by,

$$\eta_{gt} = \frac{c_p(T_3 - T_4) - c_p(T_2 - T_1)}{c_p(T_3 - T_2)} = \frac{\eta_t c_p(T_3 - T_{4s}) - c_p\,(T_{2s} - T_1)/\eta_c}{c_p(T_3 - T_1) - c_p\,(T_{2s} - T_1)/\eta_c}$$

$$\therefore \qquad \eta_{gt} = \frac{\eta_t\,\theta(1 - 1/\tau) - (\tau - 1)/\eta_c}{(\theta - 1) - (\tau - 1)/\eta_c}$$

Cross multiplying,

$$\eta_{gt}\left[\eta_c(\theta - 1) - (\tau - 1)\right] = \eta_c\eta_t\,\theta\left(1 - \frac{1}{\tau}\right) - (\tau - 1)$$

Differentiating with respect to $\tau$, keeping $\theta$, $\eta_c$ and $\eta_t$ constant,

$$\left[\eta_c(\theta - 1) - (\tau - 1)\right]\frac{\partial\eta_{gt}}{\partial\tau} - \eta_{gt} = \frac{\eta_c\eta_t\,\theta}{\tau^2} - 1$$

Maximum when $\partial\eta_{gt}/\partial\tau = 0$. Hence,

$$\tau_\eta^2 = \frac{\eta_c\,\eta_t\,\theta}{1 - \eta_{gt}^*}$$

This is an implicit expression for $\tau_\eta$ because $\eta_{gt}^*$ is itself a function of $\tau_\eta$. An explicit expression can be obtained by substituting the expression derived previously for $\eta_{gt}$. This gives a quadratic equation for $\tau_\eta$ but the solution is rather messy and not particularly informative. Given the approximate nature of the analysis, it is more instructive to write,

$$\frac{r_w}{r_\eta} = \left(\frac{\tau_w}{\tau_\eta}\right)^{\frac{\gamma}{\gamma - 1}} = \left(1 - \eta_{gt}^*\right)^{\gamma/2(\gamma - 1)} \approx 0.3 - 0.5$$

where $r$ is pressure ratio. Typically, the pressure ratio for maximum specific work is around 30 – 50 % of the pressure ratio for maximum cycle efficiency.

**6.** SFEE for the HRSG (turbine exit to pinch-point) with $\beta = m_s/m_g$ :

$$\beta(h_d - h_c) \;=\; c_p(T_4 - T_5) \;=\; c_p T_3 \left(\frac{T_4}{T_3} - \frac{T_5}{T_3}\right)$$

Changing the GT pressure ratio changes $\beta$ and $T_4$ only, all other variables remain fixed.

$$\therefore \qquad \frac{\partial\beta}{\partial\tau} = \frac{c_p T_3}{(h_d - h_c)}\frac{\partial(T_4/T_3)}{\partial\tau}$$

From the definition of isentropic efficiency for the turbine,

$$c_p(T_3 - T_4) \;=\; \eta_t\, c_p(T_3 - T_{4s})$$

$$\left(1 - \frac{T_4}{T_3}\right) = \eta_t\left(1 - \frac{T_{4s}}{T_3}\right) = \eta_t\left(1 - \frac{1}{\tau}\right)$$

$$\frac{\partial(T_4/T_3)}{\partial\tau} = -\frac{\eta_t}{\tau^2}$$

Hence,

$$\frac{\partial\beta}{\partial\tau} \;=\; -\frac{c_p T_1}{(h_d - h_c)}\frac{\eta_t\theta}{\tau^2}$$

Specific work output is given by,

$$w_x \;=\; w_t - w_c + \beta\, w_{st} \;=\; c_p(T_3 - T_4) \;-\; c_p(T_2 - T_1) \;+\; \beta(h_d - h_e)$$

$$=\; c_p\,\eta_t\,(T_3 - T_{4s}) \;-\; \frac{c_p\,(T_{2s} - T_1)}{\eta_c} \;+\; \beta(h_d - h_e)$$

$$\therefore \qquad \frac{w_x}{c_p T_1} \;=\; \left[\eta_t\,\theta\left(1 - \frac{1}{\tau}\right) \;-\; \frac{1}{\eta_c}(\tau - 1) \;+\; \beta\,\frac{(h_d - h_e)}{c_p T_1}\right]$$

Differentiating with respect to $\tau$ at constant $\theta$, $\eta_c$, $\eta_t$ and $(h_d - h_e)/c_p T_1$ gives,

$$\frac{\partial}{\partial\tau}\left(\frac{w_x}{c_p T_1}\right) \;=\; \frac{\eta_t\,\theta}{\tau^2} \;-\; \frac{1}{\eta_c} \;+\; \frac{(h_d - h_e)}{c_p T_1}\frac{\partial\beta}{\partial\tau}$$

Substituting the derived expression for $\partial\beta/\partial\tau$ gives,

$$\frac{\partial}{\partial\tau}\left(\frac{w_x}{c_p T_1}\right) \;=\; \frac{\eta_t\,\theta}{\tau^2} \;-\; \frac{1}{\eta_c} \;-\; \frac{(h_d - h_e)}{(h_d - h_c)}\frac{\eta_t\,\theta}{\tau^2}$$

The definitions of steam cycle efficiency and the parameter $\alpha$ are,

$$\eta_{st} \;=\; \frac{h_d - h_e}{h_d - h_a} \qquad\qquad \alpha \;=\; \frac{h_d - h_c}{h_d - h_a}$$

Both of these are fixed if the steam cycle is fixed so that,

$$\frac{\partial}{\partial \tau}\left(\frac{w_x}{c_p T_1}\right) = \frac{\eta_t \theta}{\tau^2} - \frac{1}{\eta_c} - \frac{\eta_{st}\,\eta_t\,\theta}{\alpha\,\tau^2}$$

Maximum specific work therefore corresponds to an isentropic temperature ratio given by,

$$\tau_w^2 = \eta_c\,\eta_t\,\theta\left(1 - \frac{\eta_{st}}{\alpha}\right)$$

Typically, we might have $\eta_{st} \approx$ 0.35-0.40 and $\alpha \approx$0.60-0.65. Hence, $\eta_{st}/\alpha \approx$ 0.6. This implies that the pressure ratio to maximise specific work for a combined-cycle is only about 20% the pressure ratio to maximise specific work for the same machine operating in simple-cycle. Such pressure ratios (around 3-4) are very low indeed.

The combined-cycle efficiency is given by,

$$\eta_{cc} = \frac{c_p(T_3 - T_4) - c_p(T_2 - T_1) + \beta(h_d - h_e)}{c_p(T_3 - T_2)}$$

$$= \frac{\eta_t c_p(T_3 - T_{4s}) - c_p(T_{2s} - T_1)/\eta_c + \beta(h_d - h_e)}{c_p(T_3 - T_1) - c_p(T_{2s} - T_1)/\eta_c}$$

$$= \frac{\eta_t\,\theta(1 - 1/\tau) - (\tau - 1)/\eta_c + \beta(h_d - h_e)/c_p T_1}{(\theta - 1) - (\tau - 1)/\eta_c}$$

Cross multiplying,

$$\eta_{cc}\left[\eta_c(\theta - 1) - (\tau - 1)\right] = \eta_c \eta_t\,\theta\left(1 - \frac{1}{\tau}\right) - (\tau - 1) + \eta_c\,\beta\frac{(h_d - h_e)}{c_p T_1}$$

Differentiating with respect to $\tau$ at constant $\theta$, $\eta_c$, $\eta_t$ and $(h_d – h_e)/c_p T_1$ gives,

$$\left[\eta_c(\theta - 1) - (\tau - 1)\right]\frac{\partial \eta_{cc}}{\partial \tau} - \eta_{cc} = \frac{\eta_c \eta_t\,\theta}{\tau^2} - 1 + \frac{\eta_c\,(h_d - h_e)}{c_p T_1}\frac{\partial \beta}{\partial \tau}$$

Substituting the expression for $\partial\beta/\partial\tau$ together with the definitions of $\eta_{st}$ and $\alpha$ gives,

$$\left[\eta_c(\theta - 1) - (\tau - 1)\right]\frac{\partial \eta_{cc}}{\partial \tau} - \eta_{cc} = \frac{\eta_c \eta_t\,\theta}{\tau^2} - 1 - \frac{\eta_{st} \eta_c \eta_t\,\theta}{\alpha\,\tau^2}$$

The maximum occurs when $\partial\eta_{cc}/\partial\tau$ = 0. Hence,

$$\tau_\eta^2 = \eta_c\,\eta_t\,\theta\left(\frac{1 - \eta_{st}/\alpha}{1 - \eta_{cc}^*}\right)$$

For $\eta_{cc}^*$ ≈ 0.55-0.60, the term in brackets is about unity. This shows that the pressure ratio corresponding to maximum combined-cycle efficiency is about the same as the pressure ratio corresponding to maximum specific work for simple-cycle operation.